Math 2601 C2
Homework 10

Below is a list of selected problems from Salas, Hille, and Etgen (the big blue book). I will select three of these problems to grade. It is in your best interest to work all of them. Please staple your work. As usual, I will be available during office hours (or a scheduled appointment if my office hours are a time conflict) and will answer questions via email (mullikin@math.gatech.edu). Homework is due Friday April 6, 2001 at 2:05 pm.

§13.4 problems 1-6,8,10,16,21

§14.1 problems 2,6,17,33,34,35

§14.2 problems 7,8,12,19,23,25

§14.3 problems 3,8,19,20,25,34,37-42

I will provide solutions to all of the even numbered problems, since the there are solutions to the odd problems in the back of the book. If you do not understand any of these solutions *please* come by my office and ask questions.

§13.4

2) $\vec{r}'(t) = \langle t^2 - 1, 2t \rangle$; $\| \vec{r}'(t) \| = t^2 + 1$; $L = int_0^2 (t^2+1)dt = \frac{14}{3}$.

4) $\vec{r}'(t) = \left\langle 1, \sqrt{2}t^{\frac{1}{2}}, t \right\rangle$; $\| \vec{r}'(t) \| = t + 1$; $L = int_0^2 (t+1)dt = 4$.

6) $\vec{r}'(t) = \left\langle \frac{1}{1+t^2}, \frac{t}{1+t^2} \right\rangle$; $\| \vec{r}'(t) \| = \frac{1}{\sqrt{1+t^2}}$; $L = \int_0^1 \frac{dt}{\sqrt{1+t^2}} = \left[\ln |t + \sqrt{t^2+1}| \right]_0^1 = \ln(1+\sqrt{2})$. Integral done using a trig substitution ($t = \tan(x)$).

8) $\vec{r}'(t) = \left\langle 1, 0, \frac{t^2}{2} - \frac{1}{2t^2} \right\rangle$; $\| \vec{r}'(t) \| = \frac{t^2}{2} + \frac{1}{2t^2}$; $L = \int_1^3 \left(\frac{t^2}{2} + \frac{1}{2t^2} \right) dt = \frac{14}{3}$.

10) $\vec{r}'(t) = \langle 3\cos(t) - 3t\sin(t), 3\sin(t) + 3t\cos(t), 4 \rangle$; $\| \vec{r}'(t) \| = \sqrt{9t^2+25}$;

$L = \int_0^4 3\sqrt{t^2 + \frac{25}{9}} dt = \left[\frac{3}{2} t \sqrt{t^2 + \frac{25}{9}} + \frac{3}{2} \cdot \frac{25}{9} \ln |t + \sqrt{t^2 + \frac{25}{9}}| \right]_0^4$

$= 26 + \frac{25}{6} \ln(5)$.

16) This problem is needed to work exercise 17, which we did in class. If you check your notes I believe you will find that by defining $\vec{r}(t) = \langle x, f(x) \rangle$ we obtain the desired result.

§14.1

2) $dom(f) = \left\{(x, y) \in \mathbb{R}^2 | xy \leqslant 1\right\}$, that is the two branches of the hyperbola $xy = 1$ and all points in between. $ran(f) = [0, \infty)$.

6) $dom(f) = \mathbb{R}^2$; $ran(f) = (-1, 1)$. Why, you may ask. Because of the following,

$$\frac{e^x - e^y}{e^x + e^y} = \frac{e^x + e^y - 2e^y}{e^x + e^y} = 1 - \frac{2}{e^{x-y} + 1}$$

34) Since we know the volume is 12 cubic somethings we have the equation $wlh = 12$. Then, just adding up the values for each side we have, $C(w, l, h) = 4wl + 2(2wh + 2lh)$. Then, using our first equation we see that $h = \frac{12}{lw}$ and so $C(l, w) = 4wl + \frac{48}{l} + \frac{48}{w}$.

§14.2

8) Hyperboloid of two sheets.

12) Elliptic paraboloid.

§14.3

8) We have the curves $e^{xy} = c$ which is a sequence of hyperbolas $xy = \ln(c)$.

20) We have the equation $x^2 + y^2 = 4$ which looks like a circle. However, recall that we are in $\mathbb{R}^3$, so we have a circle of radius 2 at each z value. Thus we obtain a circular cylinder of radius 2 going up the z-axis.

34) We start of with the equation $c = \frac{k}{\sqrt{x^2+z^2}}$. Notice that this req1uires that c be a positive number because everything on the right side of the equation is positive. Then consider the following algebraic manipulations,

$$c = \frac{k}{\sqrt{x^2 + z^2}} \Rightarrow \sqrt{x^2 + z^2} = \frac{k}{c} \Rightarrow x^2 + z^2 = \frac{k^2}{c^2}$$

So we obtain circular cylinders around the *positive* y-axis (since $c > 0$).

37-42)F; D; A; B; E; C