Math 2601 C2
Homework 11

Below is a list of selected problems from Salas, Hille, and Etgen (the big blue book), as well as some of my own. **YOU SHOULD START THIS BEFORE THURSDAY NIGHT!** I will grade the problems that I write. It is in your best interest to work all of them. All problems from the homework are fair game on exams! Please staple your work. Since you know which problems will be graded, I expect to see the solutions written *very* neatly. As usual, I will be available during office hours (or a scheduled appointment if my office hours are a time conflict) and will answer questions via email (mullikin@math.gatech.edu). Homework is due Friday April 13, 2001 at 2:05 pm.

§14.4 problems 2,3,22,28,40,43,51

§14.5 problems 1,3,5,7,9,11

§14.6 problems 1,3,9,19,22,23,26,27

§15.1 problems 1,3,5,8,25,40,41

Problem α : Slappy Funbag has just been shown some of the basic definitions from topology. Specifically open, closed, interior, and boundary. His evil instructor Mr. Nikillum has given him an assignment related to these ideas. It's your job to provide me with the answers and I will pass them on to Slappy.

Let $N := \{\left(\frac{1}{n}, \frac{m}{n}\right) \in \mathbb{R}^2 | n \in \mathbb{Z}\backslash\{0\}, m \in \mathbb{R}\}$.

Where $\mathbb{Z}\backslash\{0\} = ... - 3, -2, -1, 1, 2, 3, ...$ Notice this looks sort of line the set of all lines y=mx. Note: In your answers below I expect an explanation.

i) Is N open, closed, neither, or both?

ii) Does N have any interior points. If so, what are they.

iii) What is the boundary of N?

iv) Sketch the set N.

Problem β : Slappy Funbag was sitting through a lecture given by Mr. Nikillum on limits of functions of several variables. At some point during the class Mr. Nikillum stated that f was a function of two variables that had continuous second partials, and that $f_x(x, y) = x + y$ and $f_y(x, y) = y - x$. Much to his chagrin, Slappy caught his egregious error and made a fool of him in front of the entire class. What error did Slappy find, and how did he verify it was an error?

Problem γ : Slappy landed himself a part time job desiging thermal detection and path location equipment. That is, given moving heat source in three dimensional space, Slappy's job is to plot an interception course. Suppose that the region Slappy is working with has a temperature at each point defined by $T(x, y, z) = x^2 + xy + y^2 + z^2$. Find a path for Slappy that passes throught the point $(1, 1, 1)$. Don't forget the linear algebra we learned.

I will provide solutions to all of the even numbered problems, since the there are solutions to the odd problems in the back of the book. If you do not understand any of these solutions *please* come by my office and ask questions. Also, solutions to problems α, β, and γ were worked out in class.

§14.4

2) $g_x = 2xe^{-y}$; $g_y = -x^2e^{-y}$.

22) $h_x = f'(x)g(y)e^{f(x)g(x)}$; $h_y = f(x)g'(y)e^{f(x)g(y)}$.

28) $w_x = y\sin(z) - yz\cos(x)$; $w_y = x\sin(z) - z\sin(x)$;

$w_z = xy\cos(z) - y\sin(x)$.

40) $f_x = \frac{2xy}{z}$; $f_y = \frac{x^2}{z}$; $f_z = -\frac{x^2y}{z^2}$.

§14.6

22)a) Just taking derivatives we see that each of the mixed partials will be zero, thus they are equal.

b) Again, by taking derivatives we see that $f_x = g'(x)h(y)$ and $f_y = g(x)h'(y)$, finally the mixed partials will be $f_{xy} = g'(x)f'(x) = f_{yx}$.

c) The easy way out is to notice that the derivative of a polynomial is another polynomial, and all polynomials are continuous. Thus, f is a continuous function with continuous first and second partials, thus by citing a theorem, we see that the mixed partials must be equal. Otherwise, it suffices to check that the claim is true for any monomial, since any polynomial is a sum of monomials and differentiation is linear. Thus if each piece (monomial) gives us equality, then the whole sum (polynomial) will. Just compute the mixed partials of the function $f(x, y) = x^my^n$ and you will see that $f_{xy} = mnx^{m-1}y^{n-1} = f_{yx}$ as desired.

26)

a) As (x, y) tends to $(0, 0)$ along the x-axis, $f(x, y) = f(x, 0) = 1$ tends to 1.

As (x, y) tends to $(0, 0)$ along the line $y = x$, $f(x, y) = f(x, x) = 0$ tends to 0.

b) As (x, y) tends to $(0, 0)$ along the x-axis, $f(x, y) = f(x, 0) = 0$ tends to 0.

As (x, y) tends to $(0, 0)$ along the line $y = x$, $f(x, y) = f(x, x) = \frac{1}{2}$ tends to $\frac{1}{2}$.

§15.1

8) $\nabla f = \left\langle e^{-x}y^2, e^{-x}2xy, e^{-x}xy^2 \right\rangle$

40) With $r^n = (x^2 + y^2 + z^2)^{\frac{n}{2}}$ we have

$$\frac{\partial r^n}{\partial x} = \frac{n}{2}(x^2 + y^2 + z^2)^{\frac{n}{2}-1}(2x) = n(x^2 + y^2 + z^2)^{\frac{n-2}{2}}x = nr^{n-2}x.$$

Similarly,

$$\frac{\partial r^n}{\partial y} = nr^{n-2}y \text{and} \frac{\partial r^n}{\partial z} = nr^{n-2}z.$$

Therefore,

$$\nabla r^n = \left\langle nr^{n-2}x, nr^{n-2}y, nr^{n-2}z \right\rangle = nr^{n-2} \langle x, y, z \rangle = nr^{n-2}\vec{r}.$$