Math 2601 C2
Homework 2

Directions: Below please see the list of homework problems for this week. On Friday I will collect your solutions and select one or two problems from 12.7 to grade and I will also grade one of the three web problems. You may choose, by writing down at the top of your homework, which of the three web problems you would like for me to grade. If you do not choose I will grade the first web problem in your homework. I will collect the problems at 2:05 pm Friday January 19, 2001. *Please be sure to staple your homework and to write as neatly as possible*. If I can't read it, I can't give you any credit.

Sec 12.7: 3,5,7,15,21,23,29,37

Web Problems:

1) Let $\vec{e}_1, \vec{e}_2$, and $\vec{e}_3$ be the usual basis vectors in $\mathbb{R}^3$, and let

$$\vec{\mathbf{x}} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

i)Show that the vectors $\vec{e}_1, \vec{e}_2, \vec{e}_3$, and $\vec{\mathbf{x}}$ are linearly dependent.

ii)What conditions on x_i, x_2, and x_3 should be satisfied so that the vectors $\vec{e}_1, \vec{e}_2$, and $\vec{\mathbf{x}}$ form a basis in $\mathbb{R}^3$?

iii)Assuming $\vec{e}_1, \vec{e}_2$, and $\vec{\mathbf{x}}$ is a basis in $\mathbb{R}^3$, express $\vec{e}_3$ with respect to this basis.

2)

i)Let S be the set of vectors in $\mathbb{R}^3$ of the form $(0, a, b)$, where a and b are arbitrary numbers. Is S a linear subspace of $\mathbb{R}^3$? If so, find a basis of S and find $\dim(S)$.

ii)Let S be the set of vectors of the form $(1+t, 3t, 2-5t)$ where t is an arbitrary number (note that this is the equation of a line). Is S a linear subspace of $\mathbb{R}^3$? If so, find a basis of S and find $\dim(S)$.

3) Let V and W be linear subspaces of $\mathbb{R}^n$. Decide the following questions and prove your statements.

i)Is the set $V \cap W$ a linear subspace of $\mathbb{R}^n$?

ii)Is the set $V \setminus W$ a linear subspace of $\mathbb{R}^n$?

iii)Is the set $V + W := \{\vec{\mathbf{v}} + \vec{\mathbf{w}} \mid \vec{\mathbf{v}} \in V, \vec{\mathbf{w}} \in W\}$ a linear subspace of $\mathbb{R}^n$?