Math 2601 C2
Homework 4

Please do all three and email me if you need any assistance (mullikin@math.gatech.edu). They are to be turned in Friday Feb 2, 2001 at 2:05pm. ***HOMEWORK IS TO BE STAPLED AND SOLUTIONS ARE TO BE NEATLY WRITTEN.*** If I can't read your work, I can't give you any credit.

Problem 1 Let $\mathcal{V}$ be the space of all 2×2 real valued matrices. I.e.,

$$\mathcal{V} = \{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} | a, b, c, d \in \mathbb{R} \}$$

i) Find a basis for $\mathcal{V}$.
ii) Can you construct a linear transformation $T : \mathcal{V} \to \mathbb{R}^4$ where $Im(T) = \mathbb{R}^4$ and $Ker(T) = \vec{0}$ (Note, in this case $\vec{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$)? Such a map is called a *bijection*, and we say $\mathcal{V}$ and $\mathbb{R}^4$ are in one-to-one correspondence.

Problem 2 Let $\mathfrak{B}_1 = \{1, x, x^2\}$ be a basis for $\mathcal{P}_2$ and let $\mathfrak{B}_2 = \{1, x, x^2, x^3\}$ be a basis for $\mathcal{P}_3$. Let $T : \mathcal{P}_2 \to \mathcal{P}_3$ be the map defined by $T(p(x)) = xp(x)$.
i) Show that T is a linear transformation by using the definition.
ii) Find the matrix ${}_{\{\mathfrak{B}_2\}}M^T_{\{\mathfrak{B}_1\}}$.
iii) Find $Ker({}_{\{\mathfrak{B}_2\}}M^T_{\{\mathfrak{B}_1\}})$ and $Im({}_{\{\mathfrak{B}_2\}}M^T_{\{\mathfrak{B}_1\}})$.

Problem 3 Let $\mathfrak{B}_1 = \{1, x, x^2\}$ be a basis for $\mathcal{P}_2$, and let $\mathfrak{B}_2 = \{1, x, x^2, x^3\}$ be a basis for $\mathcal{P}_3$. Let $\heartsuit : \mathcal{P}_2 \to \mathcal{P}_3$ be the map defined by,

$$\heartsuit(p(x)) = \int_0^x p(t)dt$$

i) Find ${}_{\{\mathfrak{B}_2\}}M^\heartsuit_{\{\mathfrak{B}_1\}}$
ii) Use ${}_{\{\mathfrak{B}_2\}}M^\heartsuit_{\{\mathfrak{B}_1\}}$ to compute $\heartsuit(3x^2 + 2x + 1)$. Note, all in all you will need to compute

$$\Phi_{\{\mathfrak{B}_2\}}([{}_{\{\mathfrak{B}_2\}}M^\heartsuit_{\{\mathfrak{B}_1\}}]\Psi_{\{\mathfrak{B}_1\}}(p(x)))$$

iii) Find $Ker({}_{\{\mathfrak{B}_2\}}M^\heartsuit_{\{\mathfrak{B}_1\}})$ and $Im({}_{\{\mathfrak{B}_2\}}M^\heartsuit_{\{\mathfrak{B}_1\}})$.