Math 2601 C2
Homework 5
New And Improved!

Please do all three of the following problems and email me if you need any assistance (mullikin@math.gatech.edu). The problems are to be turned in **Monday Feb 12, 2001** at 2:05pm. Please, if your work is more than one page, find some mechanical means of maintaining some type of connectedness between the pages. Also, please write neatly. If I can't read your work, I can't give you any credit.

Problem 1 Before working this problem, please see the supplement on the web about finding inverse matrices. Consider the following 4×4 matrix.

$$A = \begin{bmatrix} 4 & -2 & 2 & 1 \\ 0 & -1 & 0 & 2 \\ 4 & -3 & 2 & 1 \\ 8 & -5 & 3 & 0 \end{bmatrix}$$

i) Either perform row operations on A or compute $detA$ to verify that A is invertible.
ii) Compute A^{-1}, and compute AA^{-1} to verify your solution is correct.

Problem 2 Let $\mathfrak{B}_1 = \{1, x, x^2\}$ be the usual basis in $\mathcal{P}_2$.
i) Check that the set $\mathfrak{B}_2 = \{x^2 - 2x, x^2 + 1, x - 1\}$ is also a basis of $\mathcal{P}_2$.
ii) Compute the change of basis matrix from $\mathfrak{B}_1$ to $\mathfrak{B}_2$ and from $\mathfrak{B}_2$ to $\mathfrak{B}_1$. Check that these two matrices are inverses of each other.

Problem 3 Let

$$\mathfrak{B}_1 = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\} = \left\{ \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \right\}$$

and

$$\mathfrak{B}_2 = \{\vec{w}_1, \vec{w}_2, \vec{w}_3\} = \left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}.$$

i) Verify that the sets are bases in $\mathbb{R}^3$.
ii) Write the vector $\vec{u} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ in the $\mathfrak{B}_1$ basis.
iii) Find the change of basis matrix from $\mathfrak{B}_1$ to $\mathfrak{B}_2$.
iv) Find the change of basis matrix from $\mathfrak{B}_2$ to $\mathfrak{B}_1$.
v) Let $\vec{a} = 2\vec{v}_1 - 2\vec{v}_2 + 3\vec{v}_3$. Write this vector in the $\mathfrak{B}_2$ basis (i.e. find $\Psi_{\mathfrak{B}_2}(\vec{a})$). Check the result by writing $\vec{a}$ in the standard basis and check that both representations really give the same vector.