Math 2601 C2
Homework 6

Please do all three of the following problems and email me if you need any assistance (mullikin@math.gatech.edu). The problems are to be turned in Friday Feb 23, 2001 at 2:05pm. Please, staple your work if it is more than one page. Also, please write neatly. If I can't read your work, I can't give you any credit.

Problem 1 Let $A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 2 & 1 \\ 3 & 2 & 3 \end{pmatrix}$.

i) Find the QR factorization of A.

ii Find the orthogonal projection P onto the column space of A.

iii) Find the orthogonal projection $P^{\perp}$ onto the complement of the column space of A.

Solution :

i) We begin by implementing the Gram-Schmidt process on the columns $\vec{a}_1, \vec{a}_2$, and $\vec{a}_3$ of A.

$$\vec{q}_1 = \frac{\vec{a}_1}{\| \vec{a}_2 \|} = \frac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Next we need to find the second vector, so we have the following,

$$\vec{w}_2 = \vec{a}_2 - (\vec{a}_2 \cdot \vec{q}_1)\vec{q}_1$$

$$= \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} - \left(0 + \frac{4}{\sqrt{14}} + \frac{6}{\sqrt{14}}\right) \begin{pmatrix} \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} \frac{10}{14} \\ \frac{20}{14} \\ \frac{30}{14} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{5}{7} \\ \frac{4}{7} \\ -\frac{1}{7} \end{pmatrix}$$

Then,

$$\vec{q}_2 = \frac{\vec{w}_2}{\parallel \vec{w}_2 \parallel} = \frac{7}{\sqrt{42}} \begin{pmatrix} -\frac{5}{7} \\ \frac{4}{7} \\ -\frac{1}{7} \end{pmatrix}$$

Lastly,

$$\vec{w}_3 = \vec{a}_3 - (\vec{a}_3 \cdot \vec{q}_2)\vec{q}_2 - (\vec{a}_3 \cdot \vec{q}_1)\vec{q}_1$$

$$= \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} - \left(-\frac{10}{\sqrt{42}} + \frac{4}{\sqrt{42}} - \frac{3}{\sqrt{42}} \right) \begin{pmatrix} -\frac{5}{\sqrt{42}} \\ \frac{4}{\sqrt{42}} \\ -\frac{1}{\sqrt{42}} \end{pmatrix} - \left(\frac{2}{\sqrt{14}} + \frac{2}{\sqrt{14}} + \frac{9}{\sqrt{14}} \right) \begin{pmatrix} \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} - \frac{9}{\sqrt{42}} \begin{pmatrix} -\frac{5}{\sqrt{42}} \\ \frac{4}{\sqrt{42}} \\ -\frac{1}{\sqrt{42}} \end{pmatrix} - \frac{13}{\sqrt{14}} \begin{pmatrix} \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Sigh, all that work and it turns out the last column is dependent on the first two. After I discovered this I went back and checked and sure enough $det(A) = 0$. So, one of the columns is non pivotal. Finally we have found,

$$Q = \begin{pmatrix} \frac{1}{\sqrt{14}} & -\frac{5}{\sqrt{42}} & \\ \frac{2}{\sqrt{14}} & \frac{4}{\sqrt{42}} & \\ \frac{3}{\sqrt{14}} & -\frac{1}{\sqrt{42}} & \end{pmatrix}$$

That's half the battle! (groan...) Fortunately, finding the entries of R only involves computing some dot products. We know that R must be upper triangular so we only need to find the entries r_{ij} where $i \leq j$. Recall the formula

$r_{ij} = \vec{q}_i \cdot \vec{a}_j$, where $\vec{q}_i$ denotes the i^{th} column of Q and a_j denotes the j^{th} column of A.

$$r_{11} = \vec{q}_1 \cdot \vec{a}_1 = \begin{pmatrix} \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \frac{14}{\sqrt{14}}$$

$$r_{12} = \vec{q}_1 \cdot \vec{a}_2 = \begin{pmatrix} \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} = \frac{10}{\sqrt{14}}$$

$$r_{13} = \vec{q}_1 \cdot \vec{a}_3 = \begin{pmatrix} \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \frac{13}{\sqrt{14}}$$

$$r_{22} = \vec{q}_2 \cdot \vec{a}_2 = \begin{pmatrix} -\frac{5}{\sqrt{42}} \\ \frac{4}{\sqrt{42}} \\ -\frac{1}{\sqrt{42}} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} = \frac{6}{\sqrt{42}}$$

$$r_{23} = \vec{q}_2 \cdot \vec{a}_3 = \begin{pmatrix} -\frac{5}{\sqrt{42}} \\ \frac{4}{\sqrt{42}} \\ -\frac{1}{\sqrt{42}} \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = -\frac{9}{\sqrt{42}}$$

Thus, we have found

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \end{pmatrix} = \begin{pmatrix} \frac{14}{\sqrt{14}} & \frac{10}{\sqrt{14}} & \frac{13}{\sqrt{14}} \\ 0 & \frac{6}{\sqrt{42}} & -\frac{9}{\sqrt{42}} \end{pmatrix}$$

Finally we have,

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 2 & 1 \\ 3 & 2 & 3 \end{pmatrix} = QR$$

$$= \begin{pmatrix} \frac{1}{\sqrt{14}} & -\frac{5}{\sqrt{42}} \\ \frac{2}{\sqrt{14}} & \frac{4}{\sqrt{42}} \\ \frac{3}{\sqrt{14}} & -\frac{1}{\sqrt{42}} \end{pmatrix} \begin{pmatrix} \frac{14}{\sqrt{14}} & \frac{10}{\sqrt{14}} & \frac{13}{\sqrt{14}} \\ 0 & \frac{6}{\sqrt{42}} & -\frac{9}{\sqrt{42}} \end{pmatrix}$$

ii) Recall that the desired projection is $P = QQ^T$, so we only need to compute a transpose and a matrix product. Indeed,

$$P = QQ^T = \begin{pmatrix} \frac{1}{\sqrt{14}} & -\frac{5}{\sqrt{42}} \\ \frac{2}{\sqrt{14}} & \frac{4}{\sqrt{42}} \\ \frac{3}{\sqrt{14}} & -\frac{1}{\sqrt{42}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{14}} & \frac{2}{\sqrt{14}} & \frac{3}{\sqrt{14}} \\ -\frac{5}{\sqrt{42}} & \frac{4}{\sqrt{42}} & -\frac{1}{\sqrt{42}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

iii) Next, remember that $P + P^{\perp} = I$. So, $P^{\perp} = I - P$. I.e.,

$$P^{\perp} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

Problem 2 Let $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

i) Find the QR factorization of B.

ii) Find the least square solution to $B\vec{x} = \vec{b}$ with

$$\vec{b} := \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$$

iii) Find the least square solution to $B\vec{x} = \vec{c}$ with

$$\vec{c} := \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

Solution :

i) As before, we need to run Gram-Schmidt on the columns of B to find the columns of Q. So, let $\vec{q}_i$ denote the i^{th} column of Q and let $\vec{b}_j$ denote the j^{th} column of B.

$$\vec{q}_1 = \frac{\vec{b}_1}{\| \vec{b}_1 \|} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{w}_2 = \vec{b}_2 - \left[\vec{b}_2 \cdot \vec{q}_1\right] \vec{q}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} - \left[\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right] \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} - [0] \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

So,

$$\vec{q}_2 = \frac{\vec{w}_2}{\parallel \vec{w}_2 \parallel} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$$

Finally,

$$\vec{w}_3 = \vec{b}_3 - \left[\vec{b}_3 \cdot \vec{q}_2\right]\vec{q}_2 - \left[\vec{b}_3 \cdot \vec{q}_1\right]\vec{q}_1$$

$$= \begin{pmatrix}1\\1\\1\\1\end{pmatrix} - \left[\begin{pmatrix}1\\1\\1\\1\end{pmatrix} \cdot \begin{pmatrix}\frac{1}{\sqrt{2}}\\0\\\frac{1}{\sqrt{2}}\\0\end{pmatrix}\right]\begin{pmatrix}\frac{1}{\sqrt{2}}\\0\\\frac{1}{\sqrt{2}}\\0\end{pmatrix} - \left[\begin{pmatrix}1\\1\\1\\1\end{pmatrix} \cdot \begin{pmatrix}0\\1\\0\\0\end{pmatrix}\right]\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$= \begin{pmatrix}1\\1\\1\\1\end{pmatrix} - \left[\frac{2}{\sqrt{2}}\right]\begin{pmatrix}\frac{1}{\sqrt{2}}\\0\\\frac{1}{\sqrt{2}}\\0\end{pmatrix} - [1]\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$= \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Then,

$$\vec{q}_3 = \frac{\vec{w}_3}{\parallel \vec{w}_3 \parallel} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

So, we have found

$$Q = \begin{pmatrix}0 & \frac{1}{\sqrt{2}} & 0\\1 & 0 & 0\\0 & \frac{1}{\sqrt{2}} & 0\\0 & 0 & 1\end{pmatrix}$$

Next we need to find the matrix R. Recall from the first problem that R is an upper triangular matrix where the nonzero elements are $r_{ij} = \vec{q}_i \cdot \vec{b}_j$ so that $A = QR$. So the dimension will work out we deduce that R must be a 3×3 matrix. So, lets find the nonzero entries.

$$r_{11} = \vec{q}_1 \cdot \vec{b}1 = 1$$

$$r_{12} = \vec{q}_1 \cdot \vec{b}2 = 0$$

$$r_{13} = \vec{q}_1 \cdot \vec{b}3 = 1$$

$$r_{22} = \vec{q}_2 \cdot \vec{b}2 = \frac{2}{\sqrt{2}}$$

$$r_{23} = \vec{q}_2 \cdot \vec{b}3 = \frac{2}{\sqrt{2}}$$

$$r_{33} = \vec{q}_3 \cdot \vec{b}3 = 1$$

So, we have

$$B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & \frac{2}{\sqrt{2}} & \frac{2}{\sqrt{2}} \\ 0 & 0 & 1 \end{pmatrix}$$

ii)

What we have here is an overdetermined system. That is, there are more equations than there are variables (since B is a 4×3 matrix). So, Theorem 6.4 in [Notes:TH] says we need to solve the system $R\vec{x} = Q^T\vec{b}$.

$$R\vec{x} = Q^T\vec{b} \Rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & \sqrt{2} & \sqrt{2} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$$

$$\begin{aligned} x_1 + x_3 &= 2 \\ \Rightarrow \quad \sqrt{2}x_2 + \sqrt{2}x_3 &= \tfrac{4}{\sqrt{2}} \\ x_3 &= 1 \end{aligned}$$

We can write this as a matrix and row reduce. Indeed,

$$\begin{pmatrix} 1 & 0 & 1 & | & 2 \\ 0 & \sqrt{2} & \sqrt{2} & | & 2\sqrt{2} \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$$

You should row reduce and verify that the solution is $\vec{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

iii) We need to do the same thing, but this time we need to solve the system $R\vec{x} = Q^T\vec{c}$.

$$R\vec{x} = Q^T\vec{c} \Rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & \sqrt{2} & \sqrt{2} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} x_1 + x_3 &= 0 \\ \Rightarrow \quad \sqrt{2}x_2 + \sqrt{2}x_3 &= 0 \\ x_3 &= 0 \end{aligned}$$

We can write this as a matrix and row reduce. Indeed,

$$\begin{pmatrix} 1 & 0 & 1 & | & 0 \\ 0 & \sqrt{2} & \sqrt{2} & | & 0 \\ 0 & 0 & 1 & | & 0 \end{pmatrix}$$

You should reduce and verify that the solution is $\vec{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

Problem 3 Let $C = \begin{pmatrix} 2 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$. Find the minimal length solution to $C\vec{x} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

Solution :
What we have here is an underdetermined system. So, we will use Theorem 6.5 in [Notes:Th] to solve this problem. The theorem tells us that if $\vec{x}$ is a solution to $C\vec{x} = \vec{b}$ then $C(P_r\vec{x}) = \vec{b}$ is a solution as well, where P_r is the projection onto the row space of C. Also, $P_r\vec{x} := \vec{x}^*$ is a minimal solution.

So, first let us find a solution to the system $C\vec{x} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$. Guess how we do that. That's right, row reduction!

$$\left(\begin{array}{cccc|c} 2 & -1 & 1 & 0 & 3 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 1 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & -1 \end{array}\right)$$

Thus, we have infinitely many solutions with two parameters. That is,

$$\vec{x} = \begin{pmatrix} 1 - \frac{2}{3}s - \frac{1}{3}t \\ -1 - \frac{1}{3}s - \frac{2}{3}t \\ s \\ t \end{pmatrix}$$

Next we need to find P_r. Recall that for getting P_r, you have to find the QR-factorization of C^T. Then $P_r = QQ^T$. Fortunately C^T only has two columns, so this shouldn't take too long.

$$\vec{q}_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

$$\vec{w}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \left[\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{2}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ 0 \end{pmatrix} \right] \begin{pmatrix} \frac{2}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \left[\frac{2}{\sqrt{6}} \right] \begin{pmatrix} \frac{2}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{3} \\ \frac{4}{3} \\ \frac{2}{3} \\ 1 \end{pmatrix}$$

So,

$$\vec{q}_2 = \frac{\vec{w}_2}{\| \vec{w}_2 \|} = \frac{3}{\sqrt{30}} \begin{pmatrix} \frac{1}{3} \\ \frac{4}{3} \\ \frac{2}{3} \\ 1 \end{pmatrix}$$

Thus,

$$P_r = QQ^T = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{3}{3\sqrt{30}} \\ -\frac{1}{\sqrt{6}} & \frac{12}{3\sqrt{30}} \\ \frac{1}{\sqrt{6}} & \frac{6}{3\sqrt{30}} \\ 0 & \frac{3}{\sqrt{30}} \end{pmatrix} \begin{pmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ \frac{3}{3\sqrt{30}} & \frac{12}{3\sqrt{30}} & \frac{6}{3\sqrt{30}} & \frac{3}{\sqrt{30}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{7}{10} & -\frac{1}{5} & \frac{2}{5} & \frac{1}{10} \\ \frac{-1}{5} & \frac{7}{10} & \frac{1}{10} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{10} & \frac{3}{10} & \frac{1}{5} \\ \frac{1}{10} & \frac{2}{5} & \frac{1}{5} & \frac{3}{10} \end{pmatrix}$$

So,

$$\vec{x}^* = \begin{pmatrix} \frac{7}{10} & -\frac{1}{5} & \frac{2}{5} & \frac{1}{10} \\ \frac{-1}{5} & \frac{7}{10} & \frac{1}{10} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{10} & \frac{3}{10} & \frac{1}{5} \\ \frac{1}{10} & \frac{2}{5} & \frac{1}{5} & \frac{3}{10} \end{pmatrix} \begin{pmatrix} 1 - \frac{2}{3}s - \frac{1}{3}t \\ -1 - \frac{1}{3}s - \frac{2}{3}t \\ s \\ t \end{pmatrix}$$

$$
= \begin{pmatrix} \frac{7}{10}1 - \frac{2}{3}s - \frac{1}{3}t - \frac{1}{5} - 1 - \frac{1}{3}s - \frac{2}{3}t + \frac{2}{5}s + \frac{1}{10}t \\ -\frac{1}{5}1 - \frac{2}{3}s - \frac{1}{3}t + \frac{7}{10} - 1 - \frac{1}{3}s - \frac{2}{3}t + \frac{1}{10}s + \frac{2}{5}t \\ \frac{2}{5}1 - \frac{2}{3}s - \frac{1}{3}t + \frac{1}{10} - 1 - \frac{1}{3}s - \frac{2}{3}t + \frac{3}{10}s + \frac{1}{5}t \\ \frac{1}{10}1 - \frac{2}{3}s - \frac{1}{3}t + \frac{2}{5} - 1 - \frac{1}{3}s - \frac{2}{3}t + \frac{1}{5}s + \frac{3}{10}t \end{pmatrix}
$$

$$
= \begin{pmatrix} (\frac{7}{10} + \frac{1}{5}) + (-\frac{14}{30} + \frac{1}{5} + \frac{2}{5})s + (-\frac{7}{30} + \frac{2}{15} + \frac{1}{10})t \\ (-\frac{1}{5} - \frac{7}{10}) + (\frac{2}{15}s - \frac{7}{30}s + \frac{1}{10}s) + (\frac{1}{15}t - \frac{14}{30}t + \frac{2}{5}t) \\ (\frac{2}{5} - \frac{1}{10}) + (-\frac{4}{15}s - \frac{1}{30}s + \frac{3}{10}s) + (-\frac{2}{15}t - \frac{2}{30}t + \frac{1}{5}t) \\ (\frac{1}{10} - \frac{2}{5}) + (-\frac{2}{30}s - \frac{2}{15}s + \frac{1}{5}s) + (-\frac{1}{30}t - \frac{4}{15}t + \frac{3}{10}t) \end{pmatrix}
$$

$$
= \begin{pmatrix} \frac{9}{10} + (-\frac{14}{30} + \frac{2}{30} + \frac{12}{30})s + (-\frac{7}{30} + \frac{4}{30} + \frac{3}{30})t \\ -\frac{9}{10} + (\frac{4}{30} - \frac{7}{30} + \frac{3}{30})s + (\frac{2}{30} - \frac{14}{30} + \frac{12}{30})t \\ \frac{3}{10} + (-\frac{8}{30} - \frac{1}{30} + \frac{9}{30})s + (-\frac{4}{30} - \frac{2}{30} + \frac{6}{30})t \\ -\frac{3}{10} + (-\frac{2}{30} - \frac{4}{30} + \frac{6}{30})s + (-\frac{1}{30} - \frac{8}{30} + \frac{9}{30})t \end{pmatrix}
$$

$$
= \begin{pmatrix} \frac{9}{10} \\ -\frac{9}{10} \\ \frac{3}{10} \\ -\frac{3}{10} \end{pmatrix}
$$

Freaky... After all that multiplication, all the paramenters went away. How 'bout that.