Math 2601 C2
Homework 7

Please do all five of the following problems and email me if you need any assistance (mullikin@math.gatech.edu). The problems are to be turned in Friday March 2, 2001 at 2:05pm. You may turn it in early (notice early $\neq$ late) if for some reason you will not be in class on Friday, although I can't imagine anyone voluntarily missing one of my stellar lectures. Please please please, staple your work if it is more than one page (please). Also, you must write neatly. If I can't read your work, I can't give you any credit.

Let $x_i(t) : \mathbb{R} \longrightarrow \mathbb{R}$ be differentiable functions for $i = 1, 2, 3$. Consider the following system of differential equations.

$$\mathbb{S} = \left\{ \begin{array}{c} x_1'(t) = 5x_1(t) + 2x_2(t) + x_3(t) \\ x_2'(t) = x_1(t) + 4x_2(t) - x_3(t) \\ x_3'(t) = -x_1(t) - 2x_2(t) + 3x_3(t) \end{array} \right\}$$

Problem 1 Find a matrix A so that $\vec{\mathbf{x}}'(t) = A\vec{\mathbf{x}}(t)$, where $\vec{\mathbf{x}}'(t) = \begin{pmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \end{pmatrix}$ and $\vec{\mathbf{x}}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$.

Problem 2 Find the roots of the characteristic polynomial [1] for A, thus obtaining the eigenvalues.

Problem 3 Find an explicit solution for the system $\mathbb{S}$, with the initial conditions $x_1(0) = 1$, $x_2(0) = 2$, and $x_3(0) = -1$.

Problem 4 Compute A^{170574}. Hint: You are *really* close to having a diagonalization for A.

Problem 5 Would you be interested in a homework assignment over spring break? The possible answers to this question are either "Boy howdy! You bet I would!" or "No thanks Mr. Sadist. I would rather have a spring *break*, not a spring *work*.".

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[1] Given a polynomial equation $0 = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, if there are any rational roots, then they will be of the form $\pm\frac{x_i}{y_j}$, where $\{x_i\}$ is the set of all divisors of $|a_0|$, and $\{y_j\}$ is the set of all divisors of $|a_n|$.