Math 2601 C2
Homework 6 + 3

Below is a list of selected problems from Salas, Hille, and Etgen (the big blue book). I will select three of these problems to grade. It is in your best interest to work all of them. Please staple your work. As usual, I will be available during office hours (or a scheduled appointment if my office hours are a time conflict) and will answer questions via email (mullikin@math.gatech.edu). Homework is due Friday March 30, 2001 at 2:05 pm.

§13.1 problems 1-8,16,18,26,30,36,40,44

§13.2 problems 1-12, 28

§13.3 problems 1,3,5,7,9,15,21,34,37,40

I will provide solutions to all of the even numbered problems, since the there are solutions to the odd problems in the back of the book. If you do not understand any of these solutions *please* come by my office and ask questions.

§13.1

2) $\vec{f}'(t) = \langle 0, 0, \sin(t) \rangle$.

4) $\vec{f}'(t) = \left\langle e^t, \frac{1}{t}, \frac{1}{1+t^2} \right\rangle$.

6) $\vec{f}'(t) = \left\langle e^t, e^t(1+t), e^t(2t+t^2) \right\rangle$.

8) $\vec{f}'(t) = \left\langle -\frac{2}{(t-1)^2}, e^{2t}(1+2t), \sec(t)\tan(t) \right\rangle$.

16) $\int_0^1 \vec{h}(t)dt = \left[\left\langle -e^{-t}(t^2+2t+2), -e^{-t}\sqrt{2}(t+1), -e^{-t} \right\rangle\right]_0^1$

$= \frac{1}{e}\left\langle (2e-5), \sqrt{2}(e-2), (e-1) \right\rangle$.

18) $\int_1^3 \vec{F}(t)dt = \left[\left\langle \ln(t), \frac{1}{2}(\ln(t))^2, -\frac{1}{2}e^{-2t} \right\rangle\right]_1^3$

$= \left\langle \ln(3), \frac{1}{2}(\ln(3))^2, \frac{1}{2}(e^{-2} - e^{-6}) \right\rangle$.

26) Circle of radius 3 oriented in the counterclockwise direction.

30) Circular helix wrapping around a cylinder of radius 2, starting at the point $(2, 0, 2\pi)$ and moving downward to the point $(2, 0, 0)$.

36) (a) $\vec{f}(t) = \langle 1 + \cos(t), \sin(t) \rangle$.

(b) $\vec{f}(t) = \langle 1 + \cos(t), -\sin(t) \rangle$.

40) $\vec{f}(t) = \langle 3 + 4t, 2, -5 + 14t \rangle$, $0 \leqslant t \leqslant 1$.

44) We can deduce from this problem that the vectors all live in $\mathbb{R}^3$ since the initial condition has three components. So, we have the following system of differential equations,

$$\vec{f}'(t) = 2\vec{f}(t), \vec{f}(0) = \langle 1, 0, -1 \rangle$$

$$\Rightarrow \begin{pmatrix} x'(t) \\ y'(t) \\ z'(t) \end{pmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$$

We know that this has a solution of the form,

$$\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = e^{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} t} \vec{f}(0)$$

Since the 3x3 matrix is diagonal we know that we have a solution of the form,

$$\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{bmatrix} e^{2t} & 0 & 0 \\ 0 & e^{2t} & 0 \\ 0 & 0 & e^{2t} \end{bmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} e^{2t} \\ 0 \\ -e^{2t} \end{pmatrix}$$

§13.2

2) $\vec{f}'(t) = \vec{b} + 2t\vec{c}$, $\vec{f}''(t) = 2\vec{c}$.

4) $\vec{f}(t) = \langle 2t^2, 0 \rangle \Rightarrow \vec{f}'(t) = \langle 4t, 0 \rangle$, $\vec{f}''(t) = \langle 4, 0 \rangle$.

6) $\vec{f}(t) = \langle 0, 0, t - t^5 \rangle \Rightarrow \vec{f}'(t) = \langle 0, 0, 1 - 5t^4 \rangle$, $\vec{f}''(t) = \langle 0, 0, -20t^3 \rangle$.

8) $\vec{f}'(t) = \langle 1, -2t, 0 \rangle \times \langle 1, t^3, 5t \rangle + \langle t, -t^2, 1 \rangle \times \langle 0, 3t^2, 5 \rangle$

$\vec{f}''(t) = \langle 0, -2, 0 \rangle \times \langle 1, t^3, 5 \rangle + \langle 2, -4t, 0 \rangle \times \langle 0, 3t^2, 5 \rangle + \langle t, -t^2, 1 \rangle \times \langle 0, 6t, 0 \rangle$.

10) $\vec{f}'(t) = \vec{g}(t^2) + t\vec{g}'(t^2)2t = \vec{g}(t^2) + 2t^2\vec{g}'(t^2)$

$\vec{f}''(t) = \vec{g}'(t^2)2t + 4t\vec{g}'(t^2) + 2t^2\vec{g}''(t^2)2t = 6t\vec{g}'(t^2) + 4t^3\vec{g}''(t^2)$.

12) $\vec{f}(t) = \langle 2e^{-2t}, 0, -2 \rangle \Rightarrow \vec{f}'(t) = \langle -4e^{-2t}, 0, 0 \rangle$, $\vec{f}''(t) = \langle 8e^{-2t}, 0, 0 \rangle$.

28) $(\vec{g} \times \vec{f})'(t) = \left[\vec{g}(t) \times \vec{f}'(t)\right] + \left[\vec{g}'(t) \times \vec{f}(t)\right]$

$= -\left[\vec{f}'(t) \times \vec{g}(t)\right] - \left[\vec{f}(t) \times \vec{g}'(t)\right]$

$= -\left[\left[\vec{f}'(t) \times \vec{g}(t)\right] + \left[\vec{f}(t) \times \vec{g}'(t)\right]\right] = -(\vec{f} \times \vec{g})'(t).$

§13.3

34) After some calculation we see that,

$$\vec{T}(t) = \frac{1}{\sqrt{a^2 \sin^2(t) + b^2 \cos^2(t)}} \langle -a \sin(t), b \cos(t) \rangle$$

.

So the question is, how can we find $\vec{N}(t)$ without doing any more computation (because it will be really bad)? We have a special case here, that is we know that our vectors must live in $\mathbb{R}^2$. So, there are only two choices for a vector that will be perpendicular to $\vec{T}(t)$. It will either be obtained by swapping the entries in $\vec{T}(t)$ and negating the first element, or by swapping the entries in $\vec{T}(t)$ and negating the second element. Since we want the vector $\vec{N}(t)$ top always point inside the ellipse we see that we have,

$$\vec{N}(t) = \frac{1}{\sqrt{a^2 \sin^2(t) + b^2 \cos^2(t)}} \langle -b \cos(t), -a \sin(t) \rangle$$

.

40) There wasn't much to this problem other than calculation.

$$\vec{T}(t) = \frac{\vec{r}'(t)}{\| \vec{r}'(t) \|} = \frac{1}{\sqrt{10}} \langle -3 \sin(3t), 1, -3 \cos(3t) \rangle$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{\| \vec{T}'(t) \|} = \frac{1}{9} \langle -9 \cos(3t), 0, 9 \sin(3t) \rangle$$

So, to find the equation of the osculating plane at the point $\vec{r}(\frac{pi}{3}) = \left(-1, \frac{\pi}{3}, 0\right)$, we will need to compute $\vec{T}(\frac{\pi}{3}) \times \vec{N}(\frac{\pi}{3})$. So, we see that,

$$\vec{T}(\frac{\pi}{3}) \times \vec{N}(\frac{\pi}{3}) = \langle 0, 1, 3 \rangle \times \langle 1, 0, 0 \rangle = \langle 0, 3, -1 \rangle$$

So, the osculating plane is defined by the equation $3(y - \frac{\pi}{3}) - z = 0$.