Practice Test
Math 2601 C2

1) Let A be the matrix representation of a linear map $T : V \longrightarrow W$, where V and W are vector spaces and,

$$A = \begin{pmatrix} 1 & -3 & 2 & 4 \\ 3 & 2 & -7 & 1 \\ 0 & -3 & -3 & 1 \\ 1 & -2 & 3 & -1 \\ 2 & -1 & 3 & 3 \end{pmatrix}$$

i) Find $Ker(A)$.

ii) Find $Im(A)$.

2) Consider the following bases for $\mathcal{P}_2$, $\mathfrak{B}_1 = \{1+x, 1+x^2, x+x^2\}$ and $\mathfrak{B}_2 = \{1+x+x^2, -x+2x^2, 3+x^2\}$

i) Find a change of basis matrix from the $\mathfrak{B}_1$ basis to the $\mathfrak{B}_2$ basis.
ii) Find $\Psi_{\mathfrak{B}_1}(3x^2 - 4x + 1)$ using matrix multiplication.

iii) Find a representation for $2(1+x+x^2) - 3(-x+2x^2) + 1(3+x^2)$ in the $\mathfrak{B}_1$ basis. Use matrices.

iv) Let $T : \mathcal{P}_2 \longrightarrow \mathcal{P}_2$ be defined by,

$$T(p(x)) = \frac{\int_0^x p(t)dt}{x}$$

Is T linear or not? Prove your assertion. If T is linear, suppose T eats vectors in the $\mathfrak{B}_1$ basis and spits out vectors in the $\mathfrak{B}_2$ basis, then find a product of three matrices which completely describes T.

3) Prove or disprove that each of the following maps is linear. Note, to show that a map $F : V \longrightarrow W$ is linear (where V and W are vector spaces) is to show that $F(\vec{x}+\vec{y}) = F(\vec{x}) + F(\vec{y})$ for all $\vec{x}, \vec{y} \in V$ and $F(\alpha\vec{x}) = \alpha F(\vec{x})$ for all scalars α and all vectors $\vec{x} \in V$. If you are trying to show that a given map is *not* linear, then you must come up with a specific counter-example.

i) $T_1 : \mathbb{R}^3 \longrightarrow \mathbb{R}$, where $T_1(\vec{x}) = -\vec{x}$.

ii) $T_2 : \mathbb{R}^3 \longrightarrow \mathbb{R}$, where $T_2(\vec{x}) = \pi$.

iii) $T_3 : \mathcal{P}_2 \longrightarrow \mathcal{P}_4$, where $T_3(p(x)) = (x + x^2)p(x)$.

iv) $T_4 : \mathbb{R}^3 \longrightarrow \mathbb{R}^6$, where $T_4(\langle x_1, x_2, x_3 \rangle) = \langle x_1, 0, x_2, 0, x_3, 0 \rangle$

v) Suppose $T : \mathbb{R}^n \longrightarrow \mathbb{R}$ is *any* linear map. Let $T_5 : \mathbb{R}^n \longrightarrow \mathbb{R}$, where $T_5(\vec{x}) = T(\vec{x}) + 1$.

4) Determine whether or not the following matrices are invertible. (You do not need to find the inverse. Unless, of course, you want to. If so, go nuts!)

$$A = \begin{pmatrix} 1 & 2-1 & \\ -2 & 1 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 27 & 347 & -69 \\ 0 & -29 & 113 \\ 0 & 0 & \pi \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & 0 & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & 0 & 0 & a_{34} & a_{35} & a_{36} \\ 0 & 0 & 0 & 0 & a_{45} & a_{46} \\ 0 & 0 & 0 & 0 & 0 & a_{56} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 32 & -1 & 29 & -13 \\ 1 & 0 & 0 & 0 \\ 132 & 0 & 597 & -23 \\ 15 & 0 & \frac{2}{23} & \frac{-1}{597} \end{pmatrix}$$