

**Practice Test**  
Math 2601 C2

Consider the following matrices.

$$A = \begin{pmatrix} 1 & -3 & 1 \\ 3 & 2 & 2 \\ 0 & -3 & -3 \\ 1 & -2 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 1 & 1 & 0 & -2 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 5 & -2 & 1 \\ -2 & 5 & -1 \\ 1 & -1 & 8 \end{pmatrix}$$

1) Find  $QR$  decompositions for  $B$  and  $D$  using

- i) Gram-Schmidt
- ii) Householder transformations
- iii) Givens rotations

2) For the following three matrices find the projection onto the column space ( $P$ ) and the projection onto the complement of the column space ( $P^\perp$ ).

- i)  $A$
- ii)  $C$
- iii)  $D$

3) Find least squares solutions for,

i)  $A\vec{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

ii)  $B\vec{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

4) Compute the following matrix norms.

- i)  $\|C\|$
- ii)  $\|D\|$

5) Solve the following systems of differential equations.

i)  $\vec{x}'(t) = C\vec{x}(t)$ , with initial condition  $\vec{x}(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ .

i)  $\vec{y}'(t) = D\vec{y}(t)$ , with initial condition  $\vec{y}(0) = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ .