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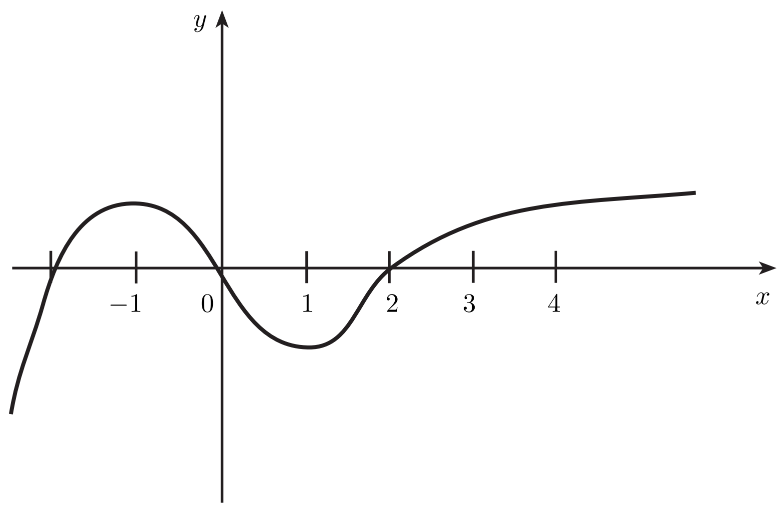
Test 2
Spring 2007
MATH 121 Section 02
March 5, 2007

Directions : You have 50 minutes to complete all 5 problems on this exam. There are a possible 100 points to be earned. You may not use your book, notes, or any graphing/programmable calculator. Please be sure to show all pertinent work. *An incorrect answer with no work will receive no credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

1. (10 points)
For the function g whose graph is given, arrange the following numbers in increasing order and explain your reasoning:

$$0 \qquad g'(-2) \qquad g'(0) \qquad g'(2) \qquad g'(4)$$

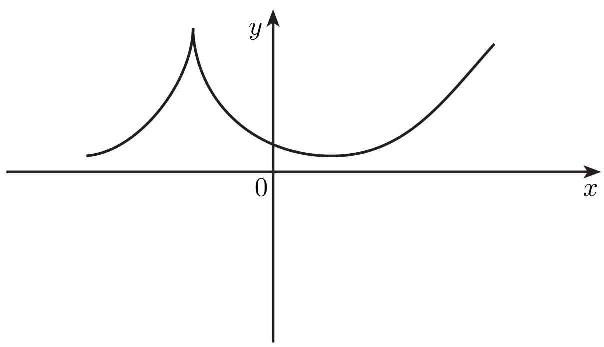


Solution :

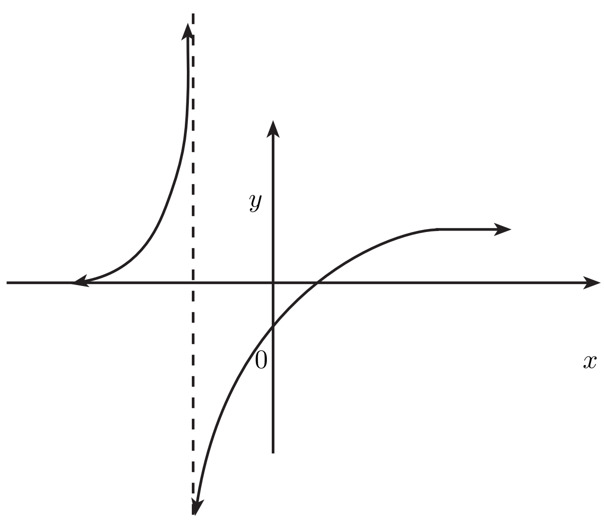
$$g'(0) \leq 0 \leq g'(4) \leq g'(2) \leq g'(-2)$$

At 0 the slope of the tangent line is negative at all other points in the list the slope is at least zero. Therefore, the smallest value is $g'(0)$. Next comes 0 since the remaining points are positive (the slope of the tangent line at -2, 2, and 4 are positive). The remaining three each have positive slope, but the slope becomes closer and closer to vertical, hence larger.

2. (10 points)
Sketch the graph of f' for the given function f below.



Solution :



3. (60 points)
Differentiate the function.

(a) $V(r) = \dfrac{4}{3}\pi r^3$ (b) $y = 4\pi^2$ (c) $g(x) = \dfrac{3x-1}{2x+1}$

(d) $f(\theta) = \dfrac{\sec(\theta)}{1+\sec(\theta)}$ (e) $y = \sin(x\cos(x))$ (f) $y = \sqrt{x+\sqrt{x}}$

Solution :

(a) $V'(r) = 4\pi r^2$

(b) $y' = 0$

(c)

$$g'(x) = \frac{2(2x+1) - 2(3x-1)}{(2x+1)^2}$$

(d)

$$f'(0) = \frac{[\sec(\theta)\tan(\theta)](1+\sec(\theta) - \sec(\theta)[\sec(\theta)\tan(\theta)]}{(1+\sec(\theta))^2}$$

(e) $y' = \cos(x\cos(x))(\cos(x) - x\sin(x))$

(f) $y' = \frac{1}{2}\left(x+\sqrt{x}\right)^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)$

4. (10 points)
Find the points on the curve $y = x^3 - x^2 - x + 1$ where the tangent line is horizontal.

Solution : We need to find all values of x where $y' = 0$. Computing a derivative we have $y' = 3x^2 - 2x - 1$. Using the quadratic formula this equation is equal to zero when

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)} = \frac{2 \pm \sqrt{16}}{6} = \frac{2 \pm 4}{6}$$

So, the tangent line is horizontal when $x = 1$ and when $x = -\frac{1}{3}$.

5. (10 points)
Use implicit differentiation to find an equation of the tangent line to the curve $x^2 + xy + y^2 = 3$ at the point $(1, 1)$.

Solution : We start by computing the implicit derivative:

$$\begin{aligned}
&\frac{d}{dx}\left(x^2 + xy + y^2\right) = \frac{d}{dx}(3) \\
&\Rightarrow 2x\frac{dx}{dx} + \frac{dx}{dx}y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0 \\
&\Rightarrow 2x + y + \frac{dy}{dx}(x + 2y) = 0 \\
&\Rightarrow \frac{dy}{dx}(x + 2y) = -2x - y \\
&\Rightarrow \frac{dy}{dx} = \frac{-2x - y}{x + 2y}
\end{aligned}$$

At the point $(1, 1)$ the slope of the tangent line is

$$m = \frac{-2 - 1}{1 + 2} = -1.$$

The equation of the tangent line is

$$y - 1 = -1(x - 1).$$