Name: <u>Dr. Mullikin</u>

Test 1
Summer 2007
MTH121 Section 01
July 3, 2007

Directions : You have 60 minutes to complete all 5 problems on this exam. There are a possible 100 points to be earned. You may not use your book, notes, or any graphing/programmable calculator. Please be sure to show all pertinent work. *A correct answer with no work will receive very little credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

1. (10 points)
   Evaluate the limit, if it exists

   a) $\lim_{x\to -1} \dfrac{x^2-2x-3}{x+1}$ b) $\lim_{x\to 2} \dfrac{\sqrt{x+2}-2}{x-2}$

   Solution :

   a)
$$\lim_{x\to -1} \frac{x^2-2x-3}{x+1} = \lim_{x\to -1} \frac{(x+1)(x-3)}{x+1}$$
$$= \lim_{x\to -1} x-3$$
$$= -4.$$

   b)
$$\lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2} = \lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2}\left(\frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}\right)$$
$$= \lim_{x\to 2} \frac{x+2-4}{(x-2)(\sqrt{x+2}+2)}$$
$$= \lim_{x\to 2} \frac{x-2}{(x-2)(\sqrt{x+2}+2)}$$
$$= \lim_{x\to 2} \frac{1}{\sqrt{x+2}+2}$$
$$= \frac{1}{4}.$$

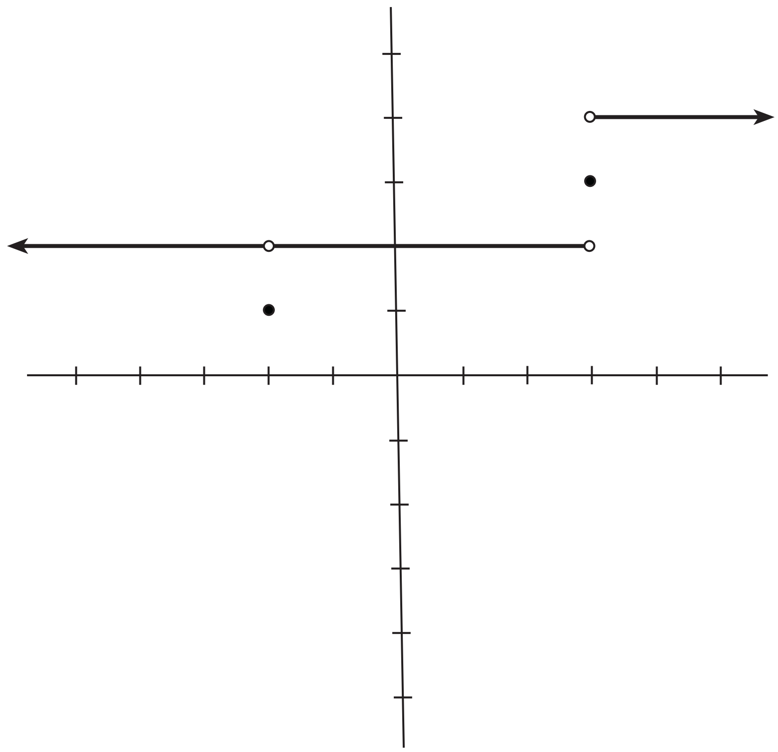
2. (20 points)
   Does there exist a real number x so that $\sin(x) = x^3 - 1$? If so, how do you know?

   Solution : Sure there is such a number. Let $f(x) = \sin(x) - x^3 + 1$ and notice that since $f(x)$ is the sum and difference of continuous functions, it is continuous. Also observe that $f(0) = 1 > 0$ and $f(\pi) = -\pi^3 + 1 < 0$. By the intermediate value theorem, there exists a number c in between 0 and π so that $f(c) = 0$. But this says that $\sin(c) - c^3 + 1 = 0$, and so $\sin(c) = c^3 - 1$ as promised.

3. (20 points)
Sketch the graph of an example of a function f that satisfies all of the given conditions.

$$\lim_{x \to 3^+} f(x) = 4,\ \lim_{x \to 3^-} f(x) = 2,\ \lim_{x \to -2} f(x) = 2,\ f(3) = 3, \text{ and} f(-2) = 1.$$

Solution :



4. (20 points)
Find the slope of the tangent to the curve $y = 2/(x+3)$ at the point where $x = a$ using the limit definition. You must use the limit definition if you wish to receive any credit.

Solution : We did this in class right before the exam. The slope of the tangent line at the point $x = a$ is the value of the derivative at $x = a$. So, we compute.

$$\lim_{h \to 0} \frac{\frac{2}{a+h+3} - \frac{2}{a+3}}{h} = \lim_{h \to 0} \frac{\frac{2(a+3)}{(a+h+3)(a+3)} - \frac{2(a+h+3)}{(a+h+3)(a+3)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2(a+3) - 2(a+h+3)}{(a+h+3)(a+3)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2a+6-2a-2h-6}{(a+h+3)(a+3)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-2h}{(a+h+3)(a+3)}}{h}$$

$$= \lim_{h \to 0} \frac{-2h}{(a+h+3)(a+3)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-2}{(a+h+3)(a+3)}$$

$$= \frac{-2}{(a+3)^2}.$$

5. (30 points)
Differentiate the function.

(a) $V(r) = \frac{4}{3}\pi r^3$ (b) $y = 4\pi^2$ (c) $g(x) = \frac{3x-1}{2x+1}$

(d) $f(\theta) = \frac{\sec(\theta)}{1+\sec(\theta)}$ (e) $y = \sin(x\cos(x))$ (f) $y = \sqrt{x+\sqrt{x}}$

Solution :

a)

$$V'(r) = 3\left(\frac{4}{3}\right)\pi r^2 = 4\pi r^2$$

b)

$$y' = 0 \ (4\pi^2 \text{ is a constant})$$

c)

$$g'(x) = \frac{(2x+1)(3)-(3x-1)(2)}{(2x+1)^2} = \frac{5}{(2x+1)^2}$$

d)

$$f'(\theta) = \frac{(1+\sec(\theta))\sec(\theta)\tan(\theta) - \sec(\theta)(\sec(\theta)\tan(\theta)}{(1+\sec(\theta))^2} = \frac{1}{(1+\sec(\theta))^2}$$

e)

$$y' = \cos(x\cos(x))\left[\cos(x) - x\sin(x)\right]$$

f)

$$y' = \frac{1}{2}\left(x + x^{1/2}\right)^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)$$