## MATH 321 Section 01 Homework 2

Below is a list of problems that I will collect Friday February 2. You should write up solutions carefully and neatly and *staple your work*. All problems from the homework are fair game on the exams! You are encouraged to work in groups, but you must write up your own solutions. I will be available during office hours for help.

(1) Show that if A is a square matrix and r and s are integers, then

$$A^r A^s = A^{r+s}$$
 and  $(A^r)^s = A^{rs}$ .

- (2) Show that if A is an invertible matrix, then

  - (a) A<sup>-1</sup> is invertible and (A<sup>-1</sup>)<sup>-1</sup> = A.
    (b) A<sup>n</sup> is invertible and (A<sup>n</sup>)<sup>-1</sup> = (A<sup>-1</sup>)<sup>n</sup> for n = 0, 1, 2, ...
  - (c) For any nonzero scalar k, the matrix kA is invertible and  $(kA)^{-1} = \frac{1}{kA^{-1}}$ .
- (3) Show that if A is an invertible matrix, then  $A^T$  is also invertible and

$$\left(A^{T}\right)^{-1} = \left(A^{-1}\right)^{T}$$

(4) Prove that the  $2 \times 2$  matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is invertible if  $ad - bc \neq 0$ , in which case the inverse is given by the formula

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} \frac{d}{ad - bc} & -\frac{b}{ad - bc} \\ -\frac{c}{ad - bc} & \frac{a}{ad - bc} \end{bmatrix}.$$

- (5) Let A and B be square matrices of the same size. Is  $(AB)^2 = A^2B^2$  a valid matrix identity? Justify your answer.
- (6) Suppose that  $d_1 d_2 d_3 \neq 0$  and consider the diagonal matrix

$$D = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}.$$

Compute  $D^{-1}$ .

(7) Consider the matrices

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 2 & -7 & 3 \end{bmatrix}$$

Find elementary matrices  $E_1$ ,  $E_2$ ,  $E_3$ , and  $E_4$  such that

- (a)  $E_1 A = B$ (b)  $E_2 B = A$ (c)  $E_3 A = C$
- (d)  $E_4 C = A$

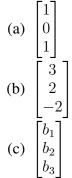
(8) Find the inverse of the given matrix if the matrix is invertible and check your answer by multiplication.

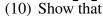
(a) 
$$\begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$$
  
(b)  $\begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}$   
(c)  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{bmatrix}$ 

(9) Consider the matrix

$$A = \begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}.$$

Solve the equation  $A\vec{x} = \vec{b}$  when  $\vec{b}$  is the matrix





	0	a	0	0	0
	b	0	c	0	0
A =	0	d	0	e	0
	0	0	f	0	g
	0	$egin{array}{c} a \\ 0 \\ d \\ 0 \\ 0 \end{array}$	0	h	0

is not invertible for any values of the entries.

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