**MATH 321 Section 01**
**Homework 4**

Below is a list of problems that I will collect Monday April 9. You should write up solutions carefully and neatly and ***staple your work***. All problems from the homework are fair game on the exams! You are encouraged to work in groups, but you must write up your own solutions. I will be available during office hours for help.

(1) Let $M_{22}$ denote the vector space of all real valued $2 \times 2$ matrices. Find two different bases for $M_{22}$.

(2) Which of the following sets of vectors in $P_2$ are linearly independent?
   (a) $2 - x + 4x^2, 3 + 6x + 2x^2, 2 + 10x - 4x^2$
   (b) $3 + x + x^2, 2 - x + 5x^2, 4 - 3x^2$
   (c) $6 - x^2, 1 + x + 4x^2$
   (d) $1 + 3x + 3x^2, x + 4x^2, 5 + 6x + 3x^2, 7 + 2x - x^2$

(3) Show that if $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_r\}$ is a linearly independent set of vectors, then so is nonempty subset of $\mathcal{B}$.

(4) Show that if $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a linearly dependent set of vectors in a vector space $\mathcal{V}$, and $\vec{v}_4$ is any vector in $\mathcal{V}$, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is also linearly dependent.

(5) Find the coordinate vector of $\vec{v}$ relative to the basis $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.
   (a) $\vec{v} = (2, -1, 3)$; $\vec{v}_1 = (1, 0, 0)$, $\vec{v}_2 = (2, 2, 0)$, $\vec{v}_3 = (3, 3, 3)$
   (b) $\vec{v} = (5, -12, 3)$; $\vec{v}_1 = (1, 2, 3)$, $\vec{v}_2 = (-4, 5, 6)$, $\vec{v}_3 = (7, -8, 9)$

(6) Find the coordinate vector of $\vec{p}$ relative to the basis $\mathcal{B} = \{\vec{p}_1, \vec{p}_2, \vec{p}_3\}$.
   (a) $\vec{p} = 4 - 3x + x^2$; $\vec{p}_1 = 1$, $\vec{p}_2 = x$, $\vec{p}_3 = x^2$
   (b) $\vec{p} = 2 - x + x^2$; $\vec{p}_1 = 1 + x$, $\vec{p}_2 = 1 + x^2$, $\vec{p}_3 = x + x^2$

(7) Determine the bases for the following subspaces of $\mathbb{R}^3$.
   (a) the plane $3x - 2y + 5z = 0$
   (b) the plane $x - y = 0$
   (c) the line $x = 2t$, $y = -t$, $z = 4t$
   (d) all vectors of the form $(a, b, c)$ where $b = a + c$.

(8) In each part, a matrix in row-echelon form is given. By inspection, find bases for the row and column spaces of $A$.

(a) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 & -1 & 5 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(9) Find a basis for the row space, column space, and nullspace of the following matrices.

(a) $\begin{pmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{pmatrix}$

(10) Find a basis for the subspace of $\mathbb{R}^4$ spanned by the given vectors.
   (a) $\vec{v}_1 = (1, 0, 1, 1)$, $\vec{v}_2 = (-3, 3, 7, 1)$, $\vec{v}_3 = (-1, 3, 9, 3)$, $\vec{v}_4 = (-5, 3, 5, -1)$
   (b) $\vec{v}_1 = (1, -2, 0, 3)$, $\vec{v}_2 = (2, -4, 0, 6)$, $\vec{v}_3 = (-1, 1, 2, 0)$, $\vec{v}_4 = (0, -1, 2, 3)$
   (c) $\vec{v}_1 = (1, -1, 5, 2)$, $\vec{v}_2 = (-2, 3, 1, 0)$, $\vec{v}_3 = (4, -5, 9, 4)$, $\vec{v}_4 = (0, 4, 2, -3)$, $\vec{v}_5 = (-7, 18, 2, -18)$

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