The problem we had was to sketch the curve $\vec{r}(t) = 2\sinh(t)\vec{i} + 2\cosh(t)\vec{j}$ where $t \geq 0$. Possibly an easier way to think about this problem (at least for me) is to rewrite the equation as the set of all points $(2\sinh(t), 2\cosh(t))$ where $t \geq 0$. Now lets give a name to each component of this set of points. Let $x(t) = 2\sinh(t)$ and $y(t) = 2\cosh(t)$ and now we are going to begin the mathematical rigorous notion of "fiddling".

These components are the hyperbolic sine and the hyperbolic cosine respectively. It may be a safe bet that the curve sketched out may have *something* to do with a hyperbola. So, why not square one of the components and see if it is related to the other in any way. The reason to square a component is that because a hyperbola has an equation of the form $ax^2 - by^2 = 1$ which implies $x^2 = \frac{1 + by^2}{a}$. Thus, it is our hope that we end up with a similar relationship between $x(t)$ and $y(t)$.

$$
\begin{aligned}
[x(t)]^2 &= 4\sinh^2(t) \\
&= 4(\cosh^2(t) - 1) \\
&= -4(1 - \cosh^2(t)) \\
&= \frac{1 - \cosh^2(t)}{-\frac{1}{4}} \\
&= \frac{1 + \frac{-1}{4}[y(t)]^2}{-\frac{1}{4}}
\end{aligned}
$$

The only no algebraic step in the above computation is going from the first line to the second. For this I hjave used the identity $\cosh^2(t) - \sinh^2(t) = 1$.

Hussah! We now have shown that all points satisfy the equation $[x(t)]^2 = \frac{1 + b[y(t)]^2}{a}$ where $b = \frac{-1}{4}$ and $a = \frac{-1}{4}$. Thus, we have the more recognizable hyperbola equation $\frac{[x(t)]^2}{-4} - \frac{[y(t)]^2}{-4} = 1$ which yields $\frac{[y(t)]^2}{4} - \frac{[x(t)]^2}{4} = 1$ which is a hyperbola that can be "easily" sketched (don't forget we only want the curve for $t \geq 0$.