MATH 2200 Fall 2002
Homework 11

Below is a list of selected problems from Edwards & Penny. It is in your best interest to work all of the problems. All problems from the homework are fair game on exams! ***Please staple your work***. I will be available during office hours for help or by email. Homework is due Wednesday December 4, 2002 at 9:05am.

§ 4.6 Problems 1,9,15,23,30,65,66,75,77,78,79,80,81,82

§ 4.7 Problems 1,3,21,25,35,40,49

α) Let

$$f(x) = \frac{x^2 + 2x + 1}{x + 1}.$$

Find a function $g(x)$ so that

$$\lim_{x \to \infty} |f(x) - g(x)| = 0,$$

thereby showing that $f(x)$ is asymptotic to $g(x)$.

(Hint)
We now know how to find both vertical and horizontal asymptotes. There are other kinds of asymptotes that we would like to be able to classify. To determine if there are any horizontal asymptotes for the graph of a function $y = f(x)$ recall that we compute the limit as x tends to plus and minus infinity. If this value converges to a constant we say that there is a horizontal tangent line at the value of a limit. So, what happens if the limit as x tends to plus or minus infinity does not coverge to a finite limit? That means that there is no horizontal asymptote, but there can be another kind. Remember that, roughly speaking, an asymptote is something that the function starts to look like far enough out. That is, if a function has a limit of 2 as x tends to infinity, then the function "looks like" the line $y = 2$ as x gets large. For example, if $f(x) = 2x/(x + 1)$ then

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{2x}{x + 1} = 2.$$

Another way to write this is that the *difference* between $f(x)$ and 2 approaches zero as x tends to infinity. That is,

$$\lim_{x \to \infty} |f(x) - 2| = \lim_{x \to \infty} \left| \frac{2x}{x + 1} - 2 \right| = \lim_{x \to \infty} \left| \frac{-2}{x + 1} \right| = 0.$$

So, more generally suppose that a function $f(x)$ begins to "look like" another function $g(x)$ as x approaches infinity. We say that $f(x)$ is asymptotic to $g(x)$ as x tends to ∞ provided

$$\lim_{x \to \infty} |f(x) - g(x)| = 0.$$

So, if $f(x) = 2x/(x+1)$ and $g(x) = 2$ we see that this is no different than the previous case. However, if $f(x) = 2x/(x^2+1) + e^x$ then notice that the fractional part will vanish as x tends to infinity and the function will "look like", i.e., be asymptotic to, the function $g(x) = e^x$. To prove this note that,

$$\lim_{x\to\infty} |f(x) - g(x)| = \lim_{x\to\infty} \left| \left(\frac{2x}{x^2+1} + e^x \right) - e^x \right| = \lim_{x\to\infty} \left| \frac{2x}{x^2+1} \right| = 0.$$

I will grade problems §4.6 #30, §4.6 #66, §4.7 #40, and α. Up to 10 points will be awarded according to how many of the other problems are completed. Please write up problems that are to be graded seperately and neatly. You may lose points if these two problems are messy.