MATH 2200 Fall 2002
Homework 4
Selected Solutions

§ 3.1 # 9) If $x = -5y^2 + 17y + 300$, find dx/dy.

Solution : There's not much to be said. We just need to compute the derivative of a polynomial.

$$\frac{dx}{dy} = \frac{d}{dy}\left(-5y^2 + 17y + 300\right) = -5(2)y + 17 = -10y + 17.$$

§ 3.1 # 26) The height $y(t)$ (in feet at time t seconds) of a ball thrown vertically upward is given by $y(t) = -16t^2 + 160t$. Find the maximum height that the ball obtains.

Solution : We can see from the sketch below that the maximum will occur where the derivative vanishes (the point at which the tangent line is horizontal).

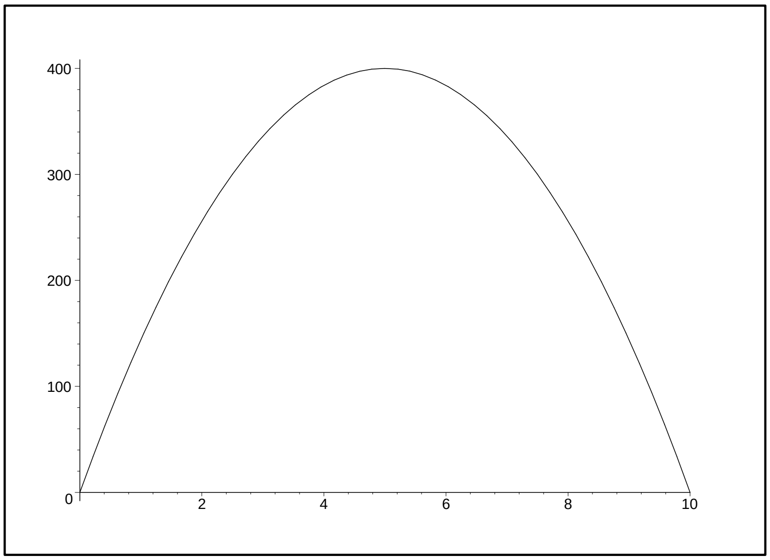

Figure 1: $y(t) = -16t^2 + 160t$

So, we only need to compute a derivative, solve for when it will be zero, and evaluate the height at the time at which the derivative vanishes. Here we go,

$$\frac{dy}{dt} = -32t + 160.$$

Now we need to find out where this is zero,

$$-32t + 160 = 0 \Rightarrow t = \frac{160}{32} = 5.$$

Finally, to find the height achieved we need to evaluate the original function at $t = 5$. So, the maximum height obtained will be $y(5) = -16(5^2) + 160(5) = 400$ feet.

§ 3.1 # 32) Match the given graph of the function f with that of its derivative. Sorry, but you will have to look in the book to follow along with this problem.

Solution : We need to look at slopes of tangent lines in order to determine what the derivative will look like. The function that was given to us appears to be increasing up until about $x = -1.5$, then it begins to decrease from $x = -1.5$ to $x = 1.5$, and then it increases after $x = 1.5$. So, we know that since the function changes from increasing to decreasing at $x = -1.5$ and $x = 1.5$ that the derivative has to change sign at these points. Thus, we will need to find a graph that passes through the x-axis at $x = -1.5$ and $x = 1.5$. The only candidates seem to be the graph in figures (b) and (f). Now we need to look a little more closely at the signs. We know that $f(x)$ is increasing up until $x = -1.5$. Therefore, it's derivative $f'(x)$ will have to be strictly positive up until $x = -1.5$. Now we are done, since (b) has this property and (f) does not. We deduce that the graph (b) is a sketch of $f'(x)$.

Just so's ya know,

#30) Figure 3.1.22 matches up with 3.1.28(c)
#31) Figure 3.1.23 matches up with 3.1.28(e)
#32) Figure 3.1.24 matches up with 3.1.28(b)
#33) Figure 3.1.25 matches up with 3.1.28(f)
#34) Figure 3.1.26 matches up with 3.1.28(a)
#35) Figure 3.1.27 matches up with 3.1.28(d)

§ 3.1 # 39) A car is traveling at 100 ft/s when the driver suddenly applies the brakes (x =0, t = 0). The position function of the skidding car is $x(t) = 100t - 5t^2$. How far and for how long does the car skid before it comes to a stop?

Solution : Well, let's get a picture first. (Thanks Maple!)

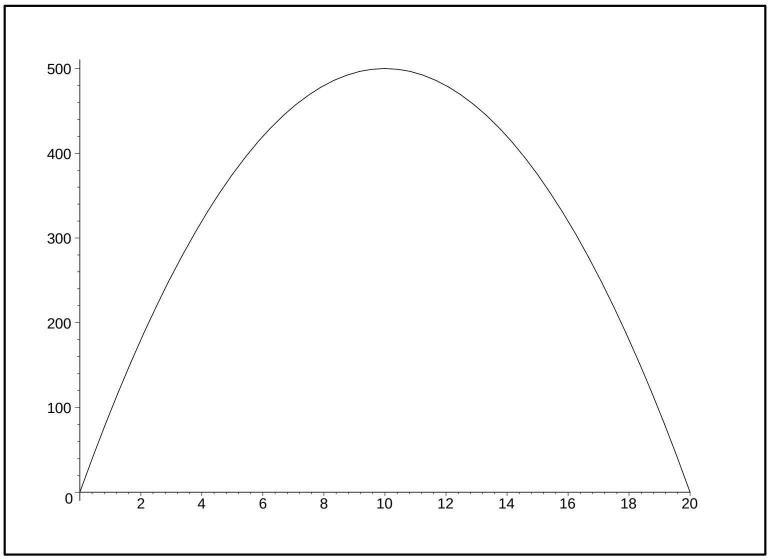

Figure 2: $x(t) = 100t - 5t^2$

It may be tempting to say that the car will stop when the curve hits the x-axis. Keep in mind though, this is a graph of the car's position, not its velocity. So, let's think a little about the physics involved in this problem. Is it likely that the car will start traveling backwards after it skids to a stop? I think not, so we need to restrict the domain of our function. The function is really only useful for us from the time $t = 0$ until the time at which the car stops entirely. Lets call the time the car stops skidding t_{end}. Let's sketch another graph that reflects our decision on the domain.

So, what happens to the graph $x(t)$ at time close to t_{end}? Well, we know that the car's velocity will decrease to zero at t_{end}. So, since velocity is the first derivative of position, we know that there will be a horizontal tangent line at t_{end}. That is, we need to find when $x'(t) = 0$ which will give us t_{end}. Then to find the distance the car skidded we need to evaluate $x(t_{\text{end}})$.

So, $x'(t) = 100 - 10t$, so $x'(t) = 0$ when $x = 10$. Therefore, $t_{\text{end}} = 10$, and the distance the car skidded will be exactly $x(t_{\text{end}}) = x(10) = 100(10) - 5(10^2) = 500$ feet.

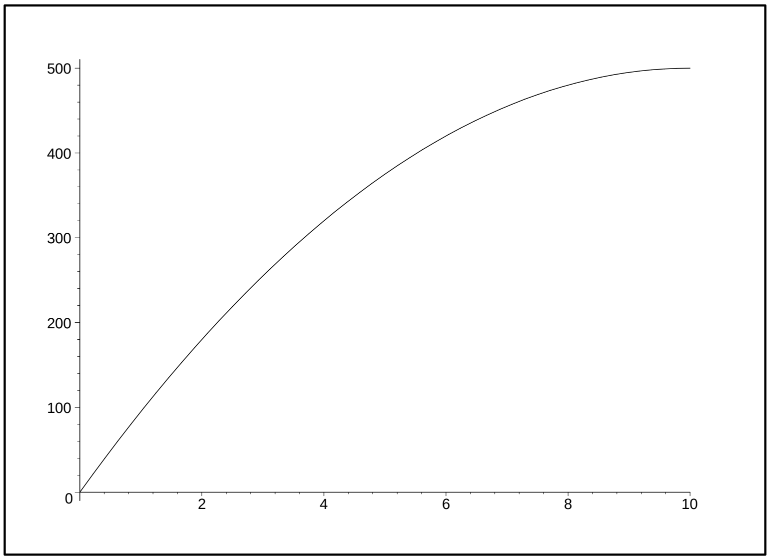

Figure 3: $x(t) = 100t - 5t^2$ for $t = [0, t_{\text{end}}]$

Notice the red herring in this problem. Nowhere did we need the fact that the car was travelling at 100 ft/sec at the instant the driver slammed on the brakes. Be careful with this type of extra information, it can lead you astray. Do notice though that the information is modelled by the position function. To say the velocity was 100 ft/sec the instant the brakes were applied is to say that the derivative evaluated at $t = 0$ had better be 100. You can check that this is the case.