MATH 2200 Fall 2002
Homework 5
Selected Solutions

§ 3.2 # 44) Write an equation for the line tangent to the curve $y = 2x - 1/x$ at the point (0.5, -1). Express the answer in the form $ax + by = c$.

Solution : Since we already know a point on the line, we only need to find the slope. So, we need to compute the rate of change of y with respect to x and evaluate it when $x = 0.5$.

$$y = 2x - 1/x = 2x - x^{-1}.$$

Therefore,

$$\frac{dy}{dx} = y' = 2 - (-1)x^{-2} = 2 + x^{-2} = 2 + \frac{1}{x^2}.$$

Evaluating this at the point $x = 0.5$ gives us the slope of the line

$$m = 2 + \frac{1}{(0.5)^2} = 2 + \frac{1}{0.25} = 2 + frac11/4 = 2 + 4 = 6.$$

So, the equation of the line will be,

$$y - (-1) = 6(x - 0.5).$$

Then, to complete the solution, we need to write this line in the form $ax + by = c$.

$$\begin{aligned} y - (-1) &= 6(x - 0.5) \\ \Rightarrow y &= 6x - 3 - 1 \\ \Rightarrow -6x + 1y &= -4. \end{aligned}$$

§ 3.2 # 59) Let $n \geq 2$ be a fixed but unspecified integer. Find the x-intercept of the line that is tangent to the curve $y = x^n$ at the point (x_0, y_0).

Solution : The first thing that needs to be done is find the equation of the tangent line. We know that a point on the line will be (x_0, y_0) where $y_0 = x_0^n$ (since it has to live on the curve $y = x^n$). So, we only need to compute the slope. This is done by computing the rate of change of y with respect to x at the point $x = x_0$. So,

$$\frac{dy}{dx} = y' = nx^{n-1}.$$

Therefore, the equation of the line will be,

$$\begin{aligned}
y - y_0 &= m(x - x_0) \\
\Rightarrow y - x_0^n &= (nx_0^{n-1})(x - x_0) \\
\Rightarrow y &= (nx_0^{n-1})(x - x_0) + x_0^n \\
\Rightarrow y &= nx_0^{n-1}x - nx_0^{n-1}x_0 + x_0^n \\
\Rightarrow y &= nx_0^{n-1}x - nx_0^n + x_0^n \\
\Rightarrow y &= nx_0^{n-1}x - (n-1)x_0^n.
\end{aligned}$$

So, to find the x-intercept we need only set $y = 0$ and solve for x. Indeed,

$$\begin{aligned}
0 &= nx_0^{n-1}x - (n-1)x_0^n \\
\Rightarrow (n-1)x_0^n &= nx_0^{n-1}x \\
\Rightarrow \frac{(n-1)x_0^n}{nx_0^{n-1}} &= x \\
\Rightarrow \frac{(n-1)x_0}{n} &= x.
\end{aligned}$$

§ 3.3 # 49) A pebble dropped into a lake creates an expanding circular ripple. Suppose that the radius of the circle is increasing at a rate of 2 in/s. At what rate is its area increasing when its radius is 10in.?

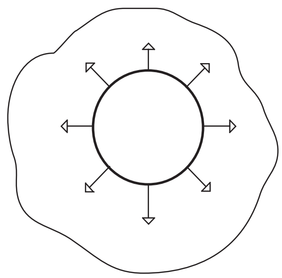

Figure 1: Expanding circular ripple in a lake.

Solution : We are trying to find the rate of increase of area, so we will need to have an area function. Fortunately, we know that the ripples look like circles and circles have a nice area formula. We know that $A(r) = \pi r^2$. We also know that the radius of the circlular ripple is increasing at a rate of 2 in/s. That is, we know $dr/dt = 2$ in/s. Since we want to know the rate of change of area we will need to compute a derivative of area. We want to know how fast the area is increasing with respect to time since the area is dependent on time. We have,

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}.$$

Then, since we know that we want the rate of change of area when the radius is 10in. Finally, since we know $dr/dt = 2$ in/s we have the rate of change of area at this instant in time is

$$2\pi(10\text{in})2\text{in/s} = 40\pi\text{in}^2/\text{s}.$$

§ 3.3 # 53) A cubical block of ice is melting in such a way that each edge decreases steadily by 2in. every hour. At what rate is it's volume decreasing when each edge is 10in. long?

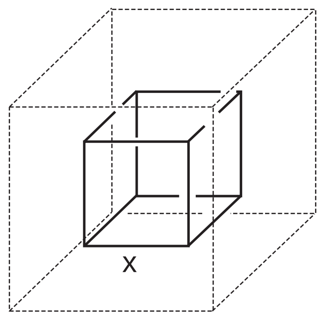

Figure 2: Cubical block.

Solution : We know that the edges are decreasing at a rate of 2in. every hour. Therefore if we denote edge length by x we know something about dx/dt. Since x is *decreasing* we should write $dx/dt = -2\text{in}/\text{hr}$. Then, we are asked to find the rate of change of volume with repect to time when $x = 10\text{in}$. So, we are looking for dV/dt where $V = x^3$ is the volume of the cube with edge length x. By the chain rule we have,

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}.$$

Finally, since we know that we want to find dV/dt at the instant in time where the edge length is 10in. we have the change in volume with respect to time is,

$$3(10\text{in})^2(-2\text{in}/\text{hr}) = -600\text{in}^3/\text{hr}.$$