MATH 2200 Fall 2002
Homework 8
Selected Solutions

§ 3.7 # 22) Find dy/dx when $y = \frac{\cos(2x)}{x}$

Solution :

$$\begin{aligned}\frac{dy}{dx} &= \frac{(-2\sin(2x))(x) - (\cos(2x))(1)}{x^2} \\ &= \frac{-2x\sin(2x) - \cos(2x)}{x^2}\end{aligned}$$

§ 3.8 # 24) Differentiate the function $f(x) = \ln\left[(1+x)^2\right]$.

Solution : I'm going to go about this two ways. The first is just brute force application of the chain rule. The second uses a property of the logarithm.

$$f'(x) = \left(\frac{1}{(1+x)^2}\right) 2(1+x) = \frac{2}{1+x}$$

Now, lets rewrite the function a little before we take the derivative.

$$f(x) = \ln\left[(1+x)^2\right] = 2\ln(1+x)$$

Then, we don't need as many applications of the chain rule.

$$f'(x) = 2\left(\frac{1}{1+x}\right) = \frac{2}{1+x}$$

α) Let

$$f(x) = \frac{e^x + e^{-x}}{2} \text{ and } g(x) = \frac{e^x - e^{-x}}{2}.$$

Prove that $f'(x) = g(x)$ and $g'(x) = f(x)$. (For the record, $f(x) = cosh(x)$ and $g(x) = sinh(x)$, the hyperbolic cosine and hyperbolic sine.)

Solution :

$$f'(x) = \frac{1}{2}\left(e^x + (-1)e^{-x}\right) = \frac{e^x - e^{-x}}{2} = g(x)$$
$$g'(x) = \frac{1}{2}\left(e^x - (-1)e^{-x}\right) = \frac{e^x + e^{-x}}{2} = f(x)$$

β) The light in an offshore lighthouse is rotating at a constant rate. Show that, as the beam of light moves down the shoreline, it moves most slowly at the point on the shore directly opposite the lighthouse.

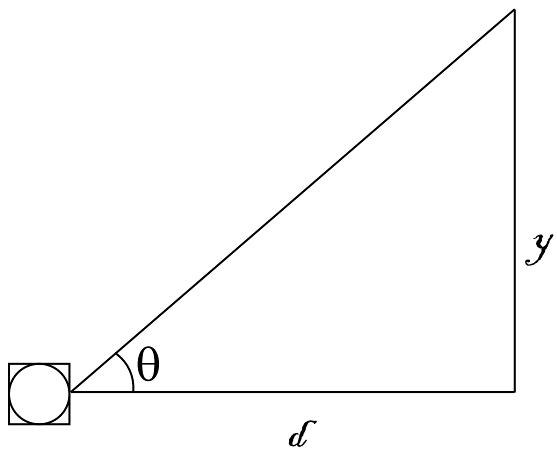

Figure 1: Lighthouse

Solution : Suppose that the lighthouse is a distance d from the shore as seen in the illustration. Then we arrive at the equation $\tan(\theta) = y/d$, and thus we see that we can express the position of the light on the shore (as a distance from the closest point on the shore to the lighthouse) as $y = d\tan(\theta)$. Since the velocity is the first derivative of position, we see that we want to show that dy/dt is smallest when $\theta = 0$.

$$\frac{dy}{dt} = d\sec^2(theta)\frac{d\theta}{dt}.$$

We know that bot d and $d\theta/dt$ are constants. So, it suffices to show that $sec^2(\theta)$ is smallest when $\theta = 0$. But this is true because $\sec(\theta) = 1/\cos(\theta)$ and $\cos(\theta)$ is largest when $\theta = 0$. Thus, $\sec^2(\theta)$ is smallest when $\theta = 0$ and we are done.