Name:____________________

Test 2 Solutions
Fall 2002
Math 2200 MWF 9:05-9:55am
October 4, 2002

Directions : You have 50 minutes to complete all 5 problems on this exam. There are a possible 100 points to be earned on this exam. You may not use programmable calculators; however you may use scientific calculators if you wish. Please be sure to show all pertinent work. *An answer with no work will receive very little credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

1. (20 points)
Compute the derivatives of the following functions with respect to x. Continue on the back of this page if you need more room.

   (a) $f(x) = 2x^2 + \frac{1}{x}$

   (b) $g(x) = \frac{2x}{3x^3+2}$

   (c) $h(x) = 2\pi + \pi^3$

   (d) $k(u) = u^2 + u$ where $u = x^2 + x$.

Solution :

a) To make things a little more nice we can rewrite the function as $f(x) = 2x^2 + x^{-1}$ and just use the power rule. Indeed, $f'(x) = 4x + (-1)x^{-2}$.

b) We need to use the quotient rule.

$$g'(x) = \frac{2(3x^3+2) - 2x(9x^2)}{(3x^3+2)^2}$$

c) h(x) is a constant function! So, $h'(x) = 0$.

d) We gotta use the chain rule. $k'(x) = k'(u)u'(x) = (2u+1)(2x+1) = (2(x^2+x)+1)(2x+1)$.

2. (20 points)
Suppose that a rocket is fired in the air and that it's distance from the surface of the Earth in miles is given by the equation $y(t) = -\frac{1}{2}t^2 + t$ for $t \geq 0$. When does the rocket hit the ground after takeoff and what maximum height did it achieve?

Solution : The rocket will hit the ground after takeoff when its height is zero. I.e., it will hit the ground again when $y(t) = 0$. So, we need to solve that equation. $y(t) = -\frac{1}{2}t^2 + t = -\frac{1}{2}t(t-2)$ so we can see that $y(t) = 0$ exactly when $t = 0$ and $t = 2$. Since the rocket was launched at time $t = 0$ we deduce that it hit the ground after takeoff at time $t = 2$.

To find the maximum height achieved we need to find when the rocket stopped going up and just started to come down. In other words, we need to know the time when the rockets velocity was zero. Since the velocity function of the rocket is given by the first derivative of its position function. So, we want to know when $y'(t) = 0$. Well, $y'(t) = -t + 1$ and this will be zero exactly when $t = 1$. So, its maximum height is $y(1) = 1/2$ miles.

3. (20 points)
Suppose that a pebble is thrown into a calm pool of water creating a circular ripple. If the *circumference* of the circle is increasing at a rate of 1 cm/sec, what is the instantaneous rate of change of area with respect to time the instant the radius is 5 cm?

Solution : First we need to identify what we know and what we want to know. We know that we are creating circular ripples. So, we are going to need some equations about circles. Next, we know that the circumference of the circle is increasing at a rate of 1 cm/sec. This means that $\frac{dC}{dt} = 1$. Then we see that the problem is asking for us to find the instantaneous rate of change of area. So we are solving for $\frac{dA}{dt}$. Furthermore, we only want to know this instantaneous rate of change when $r = 5$cm. So, lets see what $\frac{dA}{dt}$ looks like and see what we get. We know $A = \pi r^2$ so,

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}.$$

Rats! We don't know what $\frac{dr}{dt}$ is yet. So we need to get it some how. We know something about the rate of change of the circumference. So, lets use that. Since $C = 2\pi r$ we see that

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

Woo hoo! We know what $\frac{dC}{dt}$ is, so we can solve for $\frac{dr}{dt}$! Indeed,

$$1\text{cm/sec} = \frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

So, we see that $\frac{dr}{dt} = \frac{1}{2\pi}$cm/sec. Now we are in business. So, the instantaneous rate of change of area when $r = 5$ is given by,

$$2\pi r \frac{dr}{dr} = 2\pi(5\text{cm})(\frac{1}{2\pi}\text{cm/sec}) = 5\text{cm}^2/\text{sec}.$$

4. (20 points)
Consider the function $f(x)$ graphed below. Sketch a graph of $f'(x)$.

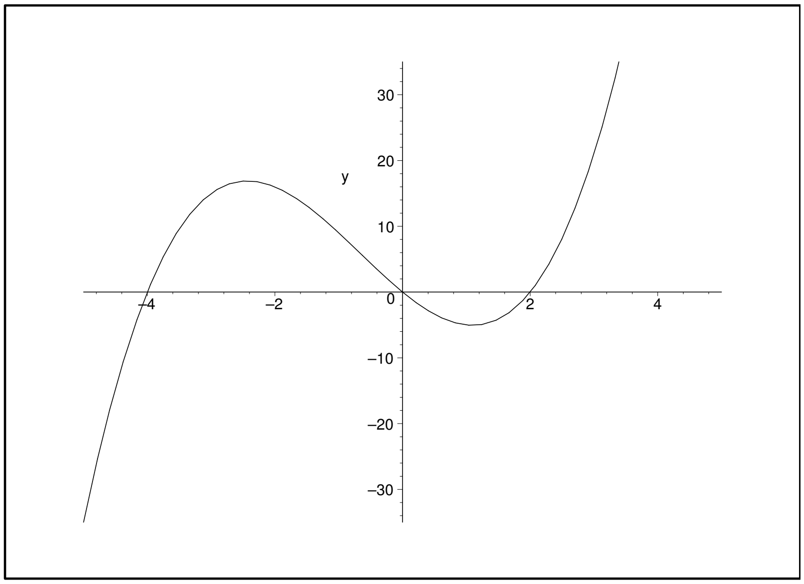

Figure 1: $f(x)$

Solution :

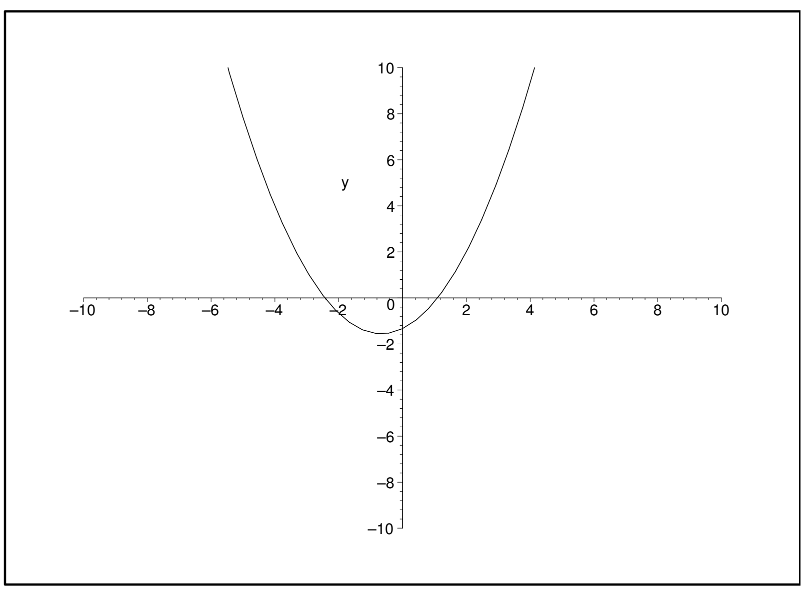

Figure 2: $f'(x)$

5. (20 points) Consider the function $g'(x)$ graphed below. Assuming that $g(0) = 0$, sketch a graph of $g(x)$.

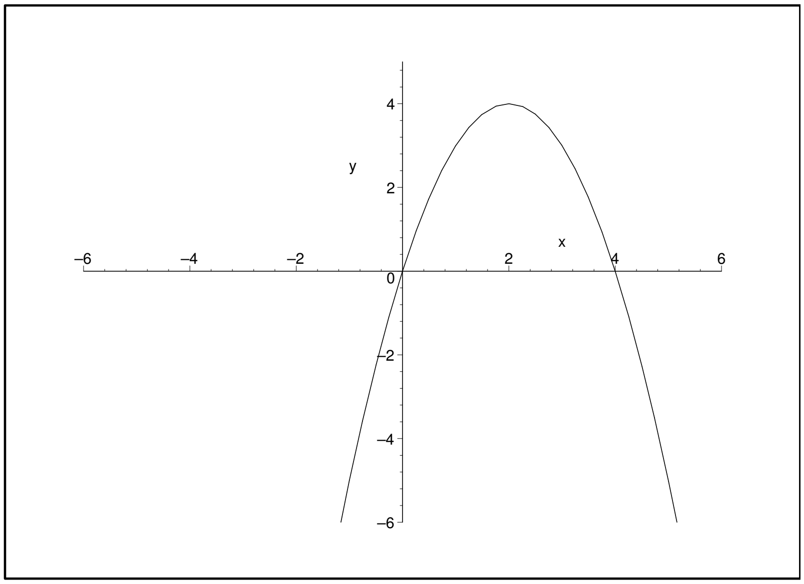

Figure 3: $g'(x)$

Solution :

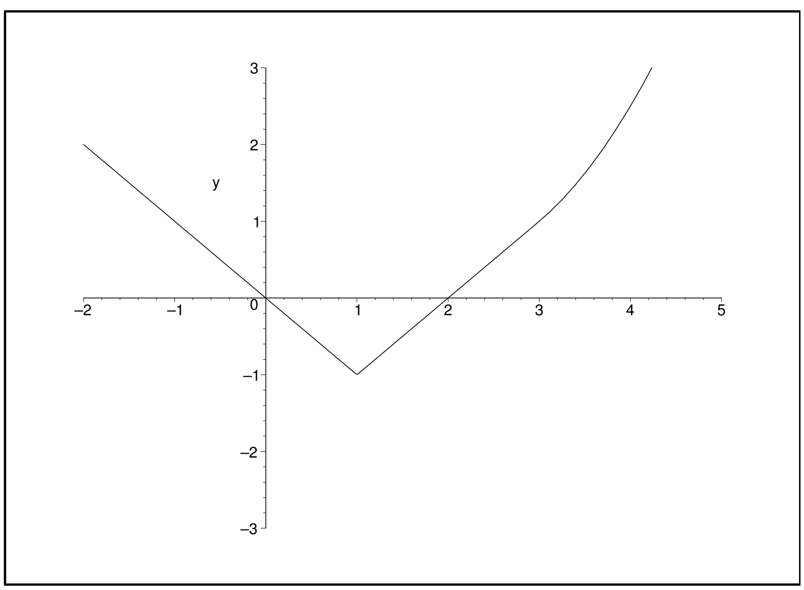

Figure 4: $f(x)$

Bonus :

(5 points)

If a car is travelling with acceleration given by $a(t) = 2t$ and it is known that the cars initial velocity (velocity at time zero) is $v(0) = 1$ and the initial position is $x(0) = -1$, what is the cars position at time t?

Solution : This problem was designed to see if you could think a little bit outside the box and try to compute some backwards derivatives (actually, they are called antiderivatives). So, we know that acceleration is the derivative of velocity which is the derivative of position. So, first we need to find the velocity. I claim that it will be $v(t) = t^2 + c_1$ where c_1 is some constant. To verify this, just compute the derivative and notice that you get $a(t)$. But what about c_1? Well, the other information given in this problem tells us that $v(0) = 1$ and $x(0) = -1$. This information is called an initial condition. So, since $v(0) = 1$ and $v(t) = t^2 + c_1$ we see that $1 = 0 + c_1$ so $c_1 = 1$. Next, we need to compute one more antiderivative. Since $v(t) = t^2 + 1$ we see that $x(t) = \frac{1}{3}t^3 + t + c_2$ (compute the derivative to check). To find c_2 we play the same game as before. We know that $x(0) = -1$ so, $-1 = \frac{1}{3}0 + 0 + c_2$. Thus, $c_2 = -1$ and the position function is $\frac{1}{3}t^3 + t - 1$.