MATH 2610 Spring 2003
Quiz 1

1. Show that $(p \wedge q) \rightarrow (p \rightarrow q)$ is a tautology using truth tables.

   Solution :

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

2. Let $S(x)$ be the predicate "x is a student," $F(x)$ the predicate "x is a faculty member," and $A(x, y)$ the predicate "x has asked y a question," where the universe of discourse consists of all people associated with your school. Use quantifiers and propositional logic notation to express the statement:

   "There is a faculty member who has never been asked a question by a student."

   Solution : We need to try to break this problem up into more manageable pieces. So, to start with look at the portion that says "there is a". This tells us that we are going to use the $\exists$ quantifier somewhere. Moreover, we are going to want to use it to say something about a faculty member. So, it seems that so far we are going to have something that looks like $\exists x \ni F(x)$. Ok, it looks like we have taken care of the faculty member. Now on to the rest of the sentence. The rest reads that he has never been asked a question by a student. Note that it does *not* say anything about the person being asked a question by another faculty member. Only *if* the person is a student do we know anything. Ok, so now we need to introduce another variable. Let's call it y. What do we know is true? We know that if y is a student then $A(y, x)$ is false provided that x is our mystery faculty person. How many students are we talking about? All of them right? So, the last bit looks like "for all people at the university, if they are a student then they have not asked faculty member x a question". That is, in notation $\forall y\, (S(y) \rightarrow \sim A(y, x))$. Combining this with the first part we have the whole statement,

$$\exists x \ni [F(x) \wedge \forall y\, (S(y) \rightarrow \sim A(y, x))]$$

3. True or False (You do not need to justify your example).

(a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}(x = y^2)$

(b) $\exists x \in \mathbb{R} \ni \forall y \in \mathbb{R}(xy = 0)$

(c) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}(x + y = 1)$

(d) $\exists x \in \mathbb{R} \ni \forall y \in \mathbb{R}(y \neq 0 \to xy = 1)$

Solution :

(a) False. Since the claim states that there exists a y for *all* x so that $x = y^2$ we only need to come up with one value for x that makes the condition $x = y^2$ fail. So, let $x = -1$. Then the claim asserts that there exists some $y \in \mathbb{R}$ so that $y^2 = -1$. Sorry, doesn't happen in $\mathbb{R}$ (although it does in $\mathbb{C}$!). So the claim is false.

(b) True. Let $x = 0$, then $0y = 0$ for any value $y \in \mathbb{R}$, therefore the claim has been proven.

(c) True. Let $y = 1 - x$.

(d) False. If this statement were true then that would imply that there exists some real number x so that $x \cdot y = 1$ for any real number $y \neq 0$. So, it has to work no matter what real number I choose for y (provided it is not zero). How about we look at what happens when we choose two different values for y. It had better be true for both values provided they are not zero. So, lets try $y_1 = 1$ and $y_2 = 2$. Then we have the equations $x \cdot 1 = 1$ and $x \cdot 2 = 1$. These equations simplify to $x = 1$ and $x = 1/2$ from which we conclude $1 = 1/2$, a contradiction. So, the claim must be false.