Name:_____

Test 1
Spring 2003
CS/MATH 2610
February 6, 2003

Directions : You have 75 minutes to complete all 6 problems on this exam. There are a possible 100 points to be earned. You may not use your book or any notes. Please be sure to show all pertinent work. *An answer with no work will receive very little credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

1. (20 points)
   Answer each of the following questions.

   (a) Define a proposition.

   (b) What does it mean for two propositions P and Q to be logically equivalent?

   (c) What is the negation of the statement $\forall x \exists y (P(x,y) \rightarrow Q(x,y))$?

   (d) Let A and B be sets. Define $A \times B$.

   (e) Let $A = \{1,2,3\}$ what is $\mathcal{P}(A)$? List the elements. ($\mathcal{P}(A)$ denotes the power set of A.)

2. (14 points)

   Let P, Q, and R be propositions. Prove or disprove that $(P \rightarrow Q) \rightarrow R$ and $P \rightarrow (Q \rightarrow R)$ are logically equivalent.

3. (16 points)
   Use Venn diagrams to sketch the following sets.

   (a) $A \cap \left(\overline{B \cap C}\right)$

   (b) $[(A \cap B) \cup (B \cap C) \cup (C \cap A)] - (A \cap B \cap C)$

4. (15 points)

Let n be an integer. Prove that if $7n + 2$ is even, then n is even.

5. (20 points)

   Let A, B, and C be sets. Prove that $A \cap \left(\overline{B \cap C} \right) = \left(A \cap \overline{B} \right) \cup \left(A \cap \overline{C} \right)$ by showing each side is a subset of the other side.(Just drawing a Venn diagram is not a proof.)

6. (15 points)

   Let $g : [0, \infty) \longrightarrow [3, \infty)$ be defined by

   $$g(x) = 3x^2 + 3.$$

   Prove that g is bijective. (Drawing a picture is not sufficient.)