(5 points) Name:_____

Test 3

Spring 2003

CS/MATH 2610

April 10, 2003

Directions : You have 75 minutes to complete all 6 problems on this exam. There are a possible 100 points to be earned. You may not use your book or any notes. Please be sure to show all pertinent work. *An answer with no work will receive very little credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

(1) (20 points)

(a) State the well ordering principle.

(b) State the principle of mathematical induction.

(c) State the binomial theorem.

(d) State the generalized pigeonhole principle.

(e) What are the formulas for $P(n, k)$ and $C(n, k)$ (where $n \geq k$)?

(2) (15 points)
Prove that $1 + 5 + 9 + \cdots + (4n - 3) = n\,(2n - 1)$ for all $n \in \mathbb{N}$.

(3) (15 points)

Let $A = \{1, 2, 3, \ldots, n\}$ and let $B = \{0, 1, 2\}$.

(a) How many functions $f : A \longrightarrow B$ exist?

(b) How many injective functions $g : A \longrightarrow B$ exist?

(c) How many functions $h : A \longrightarrow B$ exist provided $h(1) \neq$ and $h(n) \neq 0$?

(4) (15 points)

Show that if there are 100,000,000 persons employed in the U.S. who earn less than $1,000,000 in a given year, then there are at least two people that earned the same amount of money, to the penny, in that year.

(5) (15 points)

What is the coefficient of the term $x^{28}y^{72}$ in the expansion of $(x+y)^{100}$? Justify your answer with a combinatorial proof.

(6) (15 points)

In how many ways can the symbols $\{A, B, C, D, E, F\}$ be arranged provided that we insist that the symbol A come before the symbol C?