(1 point) Name:____________________

Test 2

Spring 2005

CS/MATH 2610

March 10, 2005

Directions : You have 75 minutes to complete all 7 problems on this exam. There are a possible 100 points to be earned. You may not use your book or any notes. Please be sure to show all pertinent work. *An answer with no work will receive very little credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

(1) (20 points)
Let $a, b, m \in \mathbb{Z}$ with $m > 0$. Answer each of the following questions.

(a) Define what it means to say $f(x)$ is $O(g(x))$.

(b) What is the decimal expansion of the base b number $a_k a_{k-1} \dots a_1 a_0$?

(c) Define the statement $a \mid b$.

(d) What does it mean to say the integers $a_1, a_2, \dots, a_n$ are pairwise prime?

(e) Define the statement $a \equiv b \pmod{m}$.

(2) (10 points)

Prove that $f(x)$ is $O(x)$ where

$$f(x) = \frac{2x^3 + 2x^2 + 2}{x^2 + x + 1}.$$

(3) (9 points)

Find the following products in the indicated bases without converting into decimal. You can, of course, convert into decimal to check your answer, but I would like to see the work done in the indicated bases.

(a) (Base 16)

$$\begin{array}{r} 6\mathrm{AF} \\ \underline{\times \mathrm{AF}} \end{array}$$

(b) (Base 3)

$$\begin{array}{r} 2110 \\ \underline{\times 102} \end{array}$$

(c) (Base 10)

$$\begin{array}{r} 1024 \\ \underline{\times 512} \end{array}$$

(4) (15 points)

Use the Euclidean Algorithm to compute $\gcd(159, 216)$. (You will receive very few points if you do not use the Euclidean Algorithm).

(5) (15 points)
Prove that if $a, b, c \in \mathbb{Z}$ so that $a \mid b$ and $a \mid (b + c)$, then $a \mid c$.

(6) (15 points)

Is there an integer x that leaves a remainder of 2 when divided by 4, leaves a remainder of 3 when divided by 5, and also leaves a remainder of 5 when divided by 9? If so why? (You can cite a theorem.) If there is more than one such number, what are they?

(7) (15 points)

Use mathematical induction to prove this formula for the sum of a finite number of terms of a geometric progression:

$$a + ar + ar^2 + \cdots + ar^n = \frac{ar^{n+1} - a}{r - 1}, \text{ where } r \neq 1.$$