(1 point) Name:_____

Test 3

Spring 2005

CS/MATH 2610

April 7, 2005

Directions : You have 75 minutes to complete all 7 problems on this exam. There are a possible 100 points to be earned. You may not use your book or any notes. Please be sure to show all pertinent work. *An answer with no work will receive very little credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

(1) (20 points)
Let $k, n, r \in \mathbb{Z}$ with $n > k$. Answer each of the following questions.

(a) State the Product Rule.

(b) State the Pigeonhole Principle.

(c) State the Binomial Theorem.

(d) Define an r-combination.

(e) Why is $\binom{n}{k} = \binom{n}{n-k}$?

(2) (10 points)

You, Wil Wheaton, Brent Spiner, and Michael Dorn are playing a game of cards using a standard 52 card deck. What is the total number of ways to deal five cards to all four of you? That is, how many different ways can we distribute twenty cards to four people where each person gets five cards.

(3) (9 points)

How many one-to-one functions are there from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ into the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

(4) (15 points)

Suppose x_1, x_2, x_3, and x_4 are non-negative integers. How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 9.$$

(5) (15 points)

Show that among any group of twenty six (not necessarily consecutive) integers, there are two with the same remainder when divided by 25.

(6) (15 points)

Prove that if n and k are positive integers with $n \geq k$, then

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}.$$

(7) (15 points)

Use a combinatorial proof to show that if n is a nonnegative integer, then $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$. [Hint: (1)(e).]