MATH 2610
Discrete Mathematics for Computer Science
January 13, 2005
Addendum

I need to add something to a problem that we did in class today. The problem was to show, without using truth tables, that $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ is a tautology. I unfortunately do not have notes in front of me which will allow me to reproduce the boardwork exactly, but I will try to reproduce it as well as I can.

$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

$\equiv [(p \vee q) \wedge (\sim p \vee r) \wedge (\sim q \vee r)] \to r$ (First line of Table 6 in the book)

$\equiv [(p \vee q) \wedge (r \vee \sim p) \wedge (r \vee \sim q)] \to r$ (Commutativity Law)

$\equiv [(p \vee q) \wedge (r \vee (\sim p \wedge \sim q))] \to$ (Distributivity Law backwards)

$\equiv [((p \vee q) \wedge r) \vee ((p \vee q) \wedge (\sim p \wedge \sim q))] \to r$ (Distributivity Law)

$\equiv [((p \vee q) \wedge r) \vee ((p \vee q) \wedge \sim (p \vee q))] \to r$ (Negating)

$\equiv [((p \vee q) \wedge r) \vee \mathbf{F}] \to r$ (Negation Law)

$\equiv [(p \vee q) \wedge r] \to r$ (Identity Law)

At this point I gave some painful explanation of why this must always be true. Which worked, but seemed to really be a hack job. At the end of class Mr. Mitchell approached me and asked if the following work was sufficient to complete the proof.

$\equiv [(p \vee q) \wedge r] \to r \equiv \sim [(p \vee q) \wedge r] \vee r$ (First line of Table 6 in the book)

$\equiv \sim (p \vee q) \vee \sim r \vee r$ (Negating)

$\equiv \sim (p \vee q) \vee \mathbf{T}$ (Negation)

$\equiv \mathbf{T}$ (Negation)

To answer his question, yes. Yes it is. It's not nice to show up your instructor.

–Chad

P.S. I'm just kidding about the showing up the instructor thing. If you find a better (more clever, more correct, more easily understood, etc...) method than what I present, please let me know so that I can share it with the rest of the class. Like I said, I'm not proud. I'm only interested that you learn the material.