

MATH 2610
Discrete Mathematics for Computer Science
Wednesday March, 23 2005

- (1) Show that if there are 3,276 students in a class, then at least two have last names that begin with the same letter. Are there necessarily two people with the same last name? Why or why not?
- (2) A man runs at you in the night with a bag containing red balls and green balls (it's Christmas) and *demand*s that you begin selecting balls one at a time without looking at them.
 - (a) How many balls must you select before you are sure to have at least three of the same colour?
 - (b) How many balls must you select before you are sure to have at least three green balls?
- (3) Let d be a positive integer. Show that among any group of $d + 1$ (not necessarily consecutive) integers there are two with exactly the same remainder when they are divided by d .
- (4) S'pose that there are thirteen students in a discrete mathematics course.
 - (a) Show that the class must have at least seven male students or at least seven female students.
 - (b) Show that the class must have at least nine female students or at least five male students.
- (5) Show that if there are at least 100,000,000 wage earners in the United States who earn less than 1,000,000 dollars, then there are two who earned exactly the same amount of money, to the penny, last year.
- (6) Let (x_i, y_i) , $i = 1, 2, 3, 4, 5$, be a set of five distinct points with integer coordinates in the xy -plane. Show that the midpoint of the line joining at least one pair of these points has integer coordinates. (Hint: Use the midpoint formula

$$m((x_i, y_i), (x_j, y_j)) = \left(\frac{x_i + x_j}{2}, \frac{y_i + y_j}{2} \right)$$

and the pigeon hole principle. (Hint: When is $(x_i + x_j)/2$ an integer. How many cases are there. (Hint: What do you say when you hit a golf ball?)))