MATH 2610
Discrete Mathematics for Computer Science
Thursday April, 21 2005

(1) Solve the recurrence relations together with the initial conditions given.
  (a) $a_n = a_{n-1} + 6a_{n-2}$ for $n \geq 2, a_0 = 3, a_1 = 6$
  (b) $a_n = 7a_{n-1} - 10a_{n-2}$ for $n \geq 2, a_0 = 2, a_1 = 1$
  (c) $a_n = 6a_{n-1} - 8a_{n-2}$ for $n \geq 2, a_0 = 4, a_1 = 10$
  (d) $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 2, a_0 = 4, a_1 = 1$
  (e) $a_n = a_{n-2}$ for $n \geq 2, a_0 = 5, a_1 = -1$
  (f) $a_n = -6a_{n-1} - 9a_{n-2}$ for $n \geq 2, a_0 = 3, a_1 = -3$
  (g) $a_{n+2} = -4a_{n-1} + 5a_n$ for $n \geq 0, a_0 = 2, a_1 = 8$

(2) The **Lucas numbers** satisfy the recurrence relation

$$L_n = L_{n-1} + L_{n-2},$$

and the initial conditions $L_0 = 2$ and $L_1 = 1$.
  (a) Show that $L_n = f_{n-1} + f_{n+1}$ for $n = 2, 3, 4, \ldots$, where f_n is the nth Fibonacci number.
  (b) Find an explicit formula for the Lucas numbers.

(3) Find the solution to $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$ for $n = 3, 4, 5, \ldots$, with $a_0 = 3$, $a_1 = 6$, and $a_2 = 0$.

(4) What is the general form of the particular solution of the linear nonhomogeneous recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3} + F(n)$ if
  (a) $F(n) = n^2$?
  (b) $F(n) = 2^n$?
  (c) $F(n) = n2^n$?
  (d) $F(n) = (-2)^n$?
  (e) $F(n) = n^2 2^n$?
  (f) $F(n) = n^3(-2)^n$?
  (g) $F(n) = 3$?

(5) Find all solutions of the recurrence relation $a_n = 2a_{n-1} + 2n^2$. Now find the solution with initial condition $a_1 = 4$.

(6) What beverage would you prefer (within reason) for the final exam? If coffee or tea, do you take cream and/or sugar?