MATH 2610
Discrete Mathematics for Computer Science
Thursday January, 20 2005

None of these problems will be graded. But I strongly encourage you to work all exercises other than the first one. The first exercise is called Russel's paradox and it shows some of the obnoxious problems that can show up in mathematics.

(1) Let S be the set that contains a set A if the set A does not belong to itself, so $S = \{A|A \notin A\}$. This is a little weird. It seems like it would be tricky to find any set that was not in S. It's painful to try to think of a set that contains itself. If you try long enough you may have some fellas running at you with a straight jacket, a syringe full of thorazine, and a cart to haul you away to an old dingy musty grey padded room where you will sit cold and alone staring at the pale flickering light from overhead while contemplating what a fresh warm spring breeze must feel like. But I digress.
   (a) Show that the assumption that S is a member of S leads to a contradiction.
   (b) Show that the assumption that S is not a member of S leads to a contradiction.

(2) Write the following sets by using the set notation $\{x|P(x)\}$.
   (a) The set of natural numbers strictly less than 6.
   (b) The set of integers whose square is less than 17.
   (c) $[2, 6]$
   (d) $(-1, 9]$
   (e) $[-5, -1)$
   (f) The set of rational numbers less than -1.

(3) Write the power set for each of the following sets.
   (a) $A = \{0, \heartsuit, \Delta\}$
   (b) $B = \{S, \{S\}\}$
   (c) $C = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
   (d) $D = \{1, \{2, \{3\}\}\}$

(4) True or False?
   (a) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
   (b) $\emptyset \subseteq \{\emptyset, \{\emptyset\}\}$
   (c) $\{\emptyset\} \in \{\emptyset, \{\emptyset\}\}$
   (d) $\{\emptyset\} \subseteq \{\emptyset, \{\emptyset\}\}$
   (e) $\{\{\emptyset\}\} \in \{\emptyset, \{\emptyset\}\}$
   (f) $\{\{\emptyset\}\} \subseteq \{\emptyset, \{\emptyset\}\}$
   (g) For every set A, $\emptyset \in A$.
   (h) For every set A, $\emptyset \subseteq A$.
   (i) $\{\emptyset, \{\emptyset\}\} \subseteq \{\{\emptyset, \{\emptyset\}\}\}$
   (j) $\{1, 2\} \in \{\{1, 2, 3\}, \{1, 3\}, 1, 2\}$
   (k) $\{1, 2, 3\} \subseteq \{1, 2, 3, \{4\}\}$
   (l) $\{\{4\}\} \subseteq \{1, 2, 3, \{4\}\}$