MATH 2610
Discrete Mathematics for Computer Science
Thursday January, 27 2005

These problems can be tricky. Remember that a bijection is a unction that is both one-to-one (injective) and onto (surjective). To prove that a function $f : A \longrightarrow B$is a bijection it is sufficient to prove that (a) if $f(x) = f(y)$, then $x = y$ AND (b) for all $y \in B$ there exists an $x \in A$ so that $f(x) = y$. Don't forget that the set $A \times B = \{(x, y)|x \in A, y \in B\}$ and $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$. The last problem (the one with the "*") will be considered incorrect only if it is not attempted. I don't expect you to research proof techniques or even to come up with a valid technique. I only want you to think about it and come up with *some* technique. If you want to email me to ask me questions about it I would love to answer them and discuss it with you. The idea behind this exercise is quite likely the single most important topic in this course. Okay, maybe second most important. But it's close.

(1) Let A be any set. Prove that there exists a bijection $f : A \longrightarrow A$, thus showing the astounding statement that $|A| = |A|$.

(2) Suppose that A, B, and C are sets so that $|A| = |B|$ and $|B| = |C|$. Show that there exists a bijection $h : A \longrightarrow C$. Email me if you need a hint. This is important.

(3) Let T be the set of threeven integers and let E be the set of even integers. That is let $T = \{x \in \mathbb{Z}|x = 3k \text{ for some } k \in \mathbb{Z}\}$ and let $E = \{x \in \mathbb{Z}|x = 2k \text{ for some } k \in \mathbb{Z}\}$. Prove $|E| = |T|$.

(4) Let A and B be sets. Prove that if $|A| = |\mathbb{Z}^+| = |B|$, then $|A \times B| = |\mathbb{Z}^+|$.
Hint: (2) AND $\mathbb{Q}$.

(5) Use (2) and (4) to prove that $|\mathbb{Z}^+ \times \mathbb{Z}^+ \times \mathbb{Z}^+ \times \mathbb{Z}^+| = |\mathbb{Z}^+|$.

(6) * How might you prove the more general statement $|(\mathbb{Z}^+)^n| = |\mathbb{Z}|$? Where n is any finite integer. Here $(\mathbb{Z}^+)^n = \{(x_1, x_2, \ldots, x_n)|x_1 \in \mathbb{Z}^+, x_2 \in \mathbb{Z}^+, \ldots, x_n \in \mathbb{Z}^+\}$.