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Test 3

Fall 2006

MATH 111 Section 02

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Directions : You have 50 minutes to complete all 5 problems on this exam. There are a possible 100 points to be earned. You may not use your book, notes, or any graphing/programmable calculator. Please be sure to show all pertinent work. *An incorrect answer with no work will receive no credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

1. (20 points)
   Determine the inverse of the function

$$f(x) = (2x^5 + 3)^9.$$

Solution : We write $y = (2x^5 + 3)^9$, interchange the x and y values and solve for y.

$$x = (2y^5 + 3)^9$$
$$x^{1/9} = 2y^5 + 3$$
$$x^{1/9} - 3 = 2y^5$$

$$\frac{x^{1/9} - 3}{2} = y^5$$

$$\left(\frac{x^{1/9} - 3}{2}\right)^{1/5} = y$$

2. (20 points)
Use the theorem on inverse functions to prove that f and g are inverse functions of each other, and sketch the graphs of f and g on the same coordinate plane.

$$f(x) = -x^2 + 3,\ x \geq 0$$
$$g(x) = \sqrt{3-x},\ x \leq 3.$$

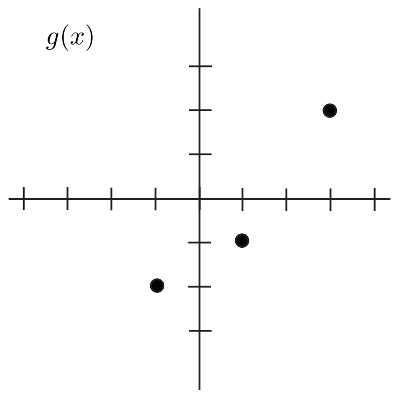
Solution : We are to show that $f(g(x)) = x$ and $g(f(x)) = x$.

$$\begin{aligned} f(g(x)) &= -\left(\sqrt{3-x}\right)^2 + 3 \\ &= -(3-x) + 3 \\ &= -3 + x + 3 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= \sqrt{3-(-x^2+3)} \\ &= \sqrt{3-x^2-3} \\ &= \sqrt{x^2} \\ &= \pm x \\ &= x \text{ (since the domain of } f(x) \text{ is } x \geq 0). \end{aligned}$$

3. (20 points)
Let $h(x) = x - 4$. Use h, the table, and the graph to evaluate the expression $g \circ h^{-1} \circ g^{-1} \circ f^{-1}(-1)$.

x	-2	0	3	7
f(x)	-1	-2	1	2



Solution : We need to compute $g\left(h^{-1}\left(g^{-1}\left(f^{-1}\left(-1\right)\right)\right)\right)$, and so we see that we will need to compute $h^{-1}(x)$. We write $y = x - 4$, interchange x and y, and solve for y.

$$\begin{aligned} x &= y - 4 \\ x + 4 &= y. \end{aligned}$$

So, we see $h^{-1}(x) = x + 4$.

$$\begin{aligned} g\left(h^{-1}\left(g^{-1}\left(f^{-1}\left(-1\right)\right)\right)\right) &= g\left(h^{-1}\left(g^{-1}\left(-2\right)\right)\right) \\ &= g\left(h^{-1}\left(-1\right)\right) \\ &= g\left(3\right) \\ &= 2. \end{aligned}$$

4. (20 points)
Solve the equation

$$4^x \cdot \left(\frac{1}{2}\right)^{3-2x} = 8 \cdot (2^x)^2 .$$

Solution : We first write everything as a power of 2.

$$\left(2^2\right)^x \cdot \left(2^{-1}\right)^{3-2x} = \left(2^3\right) \cdot (2^x)^2$$

Now we use the fact that $(a^x)^y = a^{xy}$ to rewrite the equation as

$$\left(2^{2x}\right) \cdot \left(2^{2x-3}\right) = \left(2^3\right) \cdot \left(2^{2x}\right)$$

Next we use the fact that $(a^x)(a^y) = a^{x+y}$ to get

$$2^{4x-3} = 2^{3+2x}.$$

Since the function 2^x is one-to-one we can drop the bases to get the equation that only involves the exponents and finally solve for x.

$$\begin{aligned} 4x - 3 &= 3 + 2x \\ 2x &= 6 \\ x &= 3. \end{aligned}$$

5. (20 points)
Sketch the graph and label the x-intercept and y-intercept for the function

$$f(x) = 4\left(2^{x-1}\right) - 8$$

Solution : To make my life a little easier I will first manipulate the function a little bit.

$$\begin{aligned} f(x) &= 4\left(2^{x-1}\right) - 8 \\ &= \left(2^2\right)\left(2^{x-1}\right) - 8 \\ &= 2^{x+1} - 8 \end{aligned}$$

So, the graph will be the same as the graph of the exponential function $y = 2^x$ after it has been shifted to the left by 1 unit (because of the $x+1$) and down 8 (because of the -8).