Test 4 Review
Fall 2006
MATH 111 Section 02

1. Change the following from logarithm notation to exponential notation:
   (a) $\log_2 16 = 4$ $(2^4 = 16)$
   (b) $\log_3 81 = y$ $(3^y = 81)$
   (c) $\log_e x = 3$ $(e^3 = x)$
   (d) $\log_{10} 10^x = x$ $(10^x = 10^x)$

2. Change the following from exponential notation to logarithm notation
   (a) $2^3 = 8$ $(\log_2 8 = 3)$
   (b) $e^{\ln e^x} = x$ $(\ln x = \ln e^x)$
   (c) $e^9 = x$ $(\ln x = 9)$
   (d) $10^y = x$ $(\log x = y)$

3. Find the number, if possible
   (a) $\log_2 512$ (9)
   (b) $\log_3 -3$ (no solution)
   (c) $\log_{10} 0.000001$ (-6)
   (d) $\log_2 0.125$ (-3)
   (e) $\log_1 2$ (no solution)

4. Solve the equation if possible
   (a) $\ln x^2 = -2$ $(x = \pm e^{-1})$
   (b) $\log_2 \left[(x+1)^{\frac{1}{\log_2 3}}\right] = \log_3 2$ $(x = 1)$
   (c) $\log(57x) = 2 + \log(x-2)$ $(x = {}^{200}/_{43})$
   (d) $\log x - \log(x+1) = 3\log 4$ (no solution)

5. Sketch the graphs of the following functions

   (a) $y = \log x$ (use a graphing calculator)

   (b) $y = \log(x+1)$ (same as (a) shifted to the left by 1)

   (c) $y = \log(x) + 1$ (same as (a) shifted up by 1)

   (d) $y = 2\log(x+1) - 3$ (same as (a) shifted down by 3 to the left 1 and stretched vertically by a factor of 2)

6. If the pollution of Lake Erie were stopped suddenly, it has been estimated that the level P of pollutants would decrease according to the formula

$$P = P_0 e^{-0.3821t},$$

where t is the time in years and P_0 is the pollutant level at which further pollution ceased. How many years would it take to clear 50% of the pollutants?

Solution : We want to find a value of t so that $P = 0.5P_0$.

$$0.5P_0 = P_0 e^{-0.3821t}$$
$$0.5 = e^{-0.3821t}$$
$$\ln(0.5) = -0.3821t$$

$$t = \frac{\ln(0.5)}{-0.3821}$$

$t \approx 1.81404653379$ years.

7. Verify the identity by transforming the left-hand side to the right-hand side.

   (a) $\cos^2(\alpha) - \sin^2(\alpha) = 2\cos^2(\alpha) - 1$
   Solution :
$$\begin{aligned}\cos^2(\alpha) - \sin^2(\alpha) &= \cos^2(\alpha) - (1 - \cos^2(\alpha)) \\ &= 2\cos^2(\alpha) - 1.\end{aligned}$$

   (b) $(1 + \sin(\theta))(1 - \sin(\theta)) = \sec^{-2}(\theta)$
   Solution :
$$\begin{aligned}(1 + \sin(\theta))(1 - \sin(\theta)) &= 1 - \sin^2(\theta) \\ &= \cos^2(\theta) \\ &= \sec^{-2}(\theta).\end{aligned}$$

(c) $\sec(\theta) - \cos(\theta) = \tan(\theta)\sin(\theta)$
Solution :

$$\begin{aligned}
\sec(\theta) - \cos(\theta) &= \frac{1}{\cos(\theta)} - \cos(\theta) \\
&= \frac{1}{\cos(\theta)} - \frac{\cos^2(\theta)}{\cos(\theta)} \\
&= \frac{1 - \cos^2(\theta)}{\cos(\theta)} \\
&= \frac{\sin^2(\theta)}{\cos(\theta)} \\
&= \frac{\sin(\theta)}{\cos(\theta)}\sin(\theta) \\
&= \tan(\theta)\sin(\theta)
\end{aligned}$$

(d) $\log\tan(\theta) = \log\sin(\theta) - \log\cos(\theta)$
Solution :

$$\log\tan(\theta) = \log\frac{\sin(\theta)}{\cos(\theta)} = \log\sin(\theta) - \log\cos(\theta).$$

8. Find the exact value of the following

(a) $\sin\left({}^{132\pi}/_{3}\right)$ (0)

(b) $\tan(3082\pi)$ (0)

(c) $\cot(3081\pi)$ (0)

(d) $\cos\left({}^{264\pi}/_{6}\right)$ (1)