Test 3
Fall 2006
MATH 111 Section 04
October 26, 2006

Directions : You have 75 minutes to complete all 5 problems on this exam. There are a possible 100 points to be earned. You may not use your book, notes, or any graphing/programmable calculator. Please be sure to show all pertinent work. *An incorrect answer with no work will receive no credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

1. (15 points)

   Determine the inverse of the function

   $$f(x) = (x^5 + 1)^7.$$

   Solution : We write $y = (x^5 + 1)^7$, interchange the x and y values and solve for y.

   $$x = (y^5 + 1)^7$$
   $$x^{1/7} = y^5 + 1$$
   $$x^{1/7} - 1 = y^5$$

   $$\left(x^{1/7} - 1\right)^{1/5} = y$$

2. (15 points)
   Determine the inverse of the function
   $$g(x) = \frac{2x+3}{x+3}.$$

   Solution : We write $y = {2x+3}/{x+3}$, interchange the x and y values and solve for y.

   $$x = \frac{2y+3}{y+3}$$

   $$x(y+3) = 2y+3$$

   $$xy + 3x = 2y + 3$$

   $$xy - 2y = 3 - 3x$$

   $$y(x-2) = 3 - 3x$$

   $$y = \frac{3-3x}{x-2}.$$

3. (20 points)

Use the theorem on inverse functions to prove that f and g are inverse functions of each other, and sketch the graphs of f and g on the same coordinate plane.

$$f(x) = -x^2 + 3, \ x \geq 0$$
$$g(x) = \sqrt{3 - x}, \ x \leq 3.$$

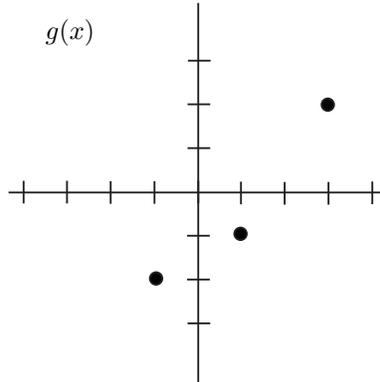**Solution :** We are to show that $f(g(x)) = x$ and $g(f(x)) = x$.

$$\begin{aligned}
f(g(x)) &= -\left(\sqrt{3-x}\right)^2 + 3 \\
&= -(3 - x) + 3 \\
&= -3 + x + 3 \\
&= x
\end{aligned}$$

$$\begin{aligned}
g(f(x)) &= \sqrt{3 - (-x^2 + 3)} \\
&= \sqrt{3 - x^2 - 3} \\
&= \sqrt{x^2} \\
&= \pm x \\
&= x \text{ (since the domain of } f(x) \text{ is } x \geq 0).
\end{aligned}$$

4. (20 points)

Let $h(x) = 2x+3$. Use h, the table, and the graph to evaluate the expression $h^{-1} \circ g \circ h \circ g^{-1} \circ f^{-1}(-1)$.

x	-2	0	3	7
f(x)	-1	-2	1	2

$g(x)$

Solution : We need to compute $h^{-1}\left(g\left(h\left(g^{-1}\left(f^{-1}(-1)\right)\right)\right)\right)$, and so we see that we will need to compute $h^{-1}(x)$. We write $y = 2x + 3$, interchange x and y, and solve for y.

$$x = 2y + 3$$
$$x - 3 = 2y$$
$${}^{x-3}/_2 = y.$$

So, we see $h^{-1}(x) = {}^{x-3}/_2$.

$$
\begin{aligned}
h^{-1}\left(g\left(h\left(g^{-1}\left(f^{-1}(-1)\right)\right)\right)\right) &= h^{-1}\left(g\left(h\left(g^{-1}(-2)\right)\right)\right) \\
&= h^{-1}\left(g\left(h\left(-1\right)\right)\right) \\
&= h^{-1}\left(g\left(1\right)\right) \\
&= h^{-1}\left(-1\right) \\
&= -2.
\end{aligned}
$$

5. (15 points)

   Solve the equation

$$4^x \cdot \left(\frac{1}{2}\right)^{3-2x} = 8 \cdot (2^x)^2.$$

Solution : We first write everything as a power of 2.

$$\left(2^2\right)^x \cdot \left(2^{-1}\right)^{3-2x} = \left(2^3\right) \cdot (2^x)^2$$

Now we use the fact that $(a^x)^y = a^{xy}$ to rewrite the equation as

$$\left(2^{2x}\right) \cdot \left(2^{2x-3}\right) = \left(2^3\right) \cdot \left(2^{2x}\right)$$

Next we use the fact that $(a^x)(a^y) = a^{x+y}$ to get

$$2^{4x-3} = 2^{3+2x}.$$

Since the function 2^x is one-to-one we can drop the bases to get the equation that only involves the exponents and finally solve for x.

$$4x - 3 = 3 + 2x$$
$$2x = 6$$
$$x = 3.$$

6. (15 points)
   Sketch the graph of the function
   $$f(x) = 4\left(2^{x-3}\right) - 3$$

   Solution : To make my life a little easier I will first manipulate the function a little bit.

   $$\begin{aligned} f(x) &= 4\left(2^{x-3}\right) - 3 \\ &= \left(2^2\right)\left(2^{x-3}\right) - 3 \\ &= 2^{x-1} - 3 \end{aligned}$$

   So, the graph will be the same as the graph of the exponential function $y = 2^x$ after it has been shifted to the right by 1 unit (because of the $x - 1$) and down 3 (because of the -3).

$(1, -2)$