

Test 4 Review
Fall 2006
MATH 111 Section 04

1. Change the following from logarithm notation to exponential notation:

- | | |
|--------------------------|-----------------|
| (a) $\log_2 16 = 4$ | $(2^4 = 16)$ |
| (b) $\log_3 81 = y$ | $(3^y = 81)$ |
| (c) $\log_e x = 3$ | $(e^3 = x)$ |
| (d) $\log_{10} 10^x = x$ | $(10^x = 10^x)$ |

2. Change the following from exponential notation to logarithm notation

- | | |
|-----------------------|---------------------|
| (a) $2^3 = 8$ | $(\log_2 8 = 3)$ |
| (b) $e^{\ln e^x} = x$ | $(\ln x = \ln e^x)$ |
| (c) $e^9 = x$ | $(\ln x = 9)$ |
| (d) $10^y = x$ | $(\log x = y)$ |

3. Find the number, if possible

- | | |
|--------------------------|------------------------|
| (a) $\log_2 512$ | (9) |
| (b) $\log_3 -3$ | (no solution) |
| (c) $\log_{10} 0.000001$ | (-6) |
| (d) $\log_2 0.125$ | (-3) |
| (e) $\log_1 2$ | (no solution) |

4. Solve the equation if possible

- | | |
|---|------------------------|
| (a) $\ln x^2 = -2$ | $(x = \pm e^{-1})$ |
| (b) $\log_2 \left[(x+1)^{\frac{1}{\log_2 3}} \right] = \log_3 2$ | $(x = 1)$ |
| (c) $\log(57x) = 2 + \log(x-2)$ | $(x = 200/43)$ |
| (d) $\log x - \log(x+1) = 3 \log 4$ | (no solution) |

5. Sketch the graphs of the following functions

- (a) $y = \log x$ (use a graphing calculator)
 (b) $y = \log(x + 1)$ (same as (a) shifted to the left by 1)
 (c) $y = \log(x) + 1$ (same as (a) shifted up by 1)
 (d) $y = 2 \log(x + 1) - 3$ (same as (a) shifted down by 3 to the left 1 and stretched vertically by a factor of 2)

6. If the pollution of Lake Erie were stopped suddenly, it has been estimated that the level P of pollutants would decrease according to the formula

$$P = P_0 e^{-0.3821t},$$

where t is the time in years and P_0 is the pollutant level at which further pollution ceased. How many years would it take to clear 50% of the pollutants?

Solution : We want to find a value of t so that $P = 0.5P_0$.

$$0.5P_0 = P_0 e^{-0.3821t}$$

$$0.5 = e^{-0.3821t}$$

$$\ln(0.5) = -0.3821t$$

$$t = \frac{\ln(0.5)}{-0.3821}$$

$$t \approx 1.81404653379 \text{ years.}$$

7. Verify the identity by transforming the left-hand side to the right-hand side.

(a) $\cos^2(\alpha) - \sin^2(\alpha) = 2\cos^2(\alpha) - 1$

Solution :

$$\begin{aligned} \cos^2(\alpha) - \sin^2(\alpha) &= \cos^2(\alpha) - (1 - \cos^2(\alpha)) \\ &= 2\cos^2(\alpha) - 1. \end{aligned}$$

(b) $(1 + \sin(\theta))(1 - \sin(\theta)) = \sec^{-2}(\theta)$

Solution :

$$\begin{aligned} (1 + \sin(\theta))(1 - \sin(\theta)) &= 1 - \sin^2(\theta) \\ &= \cos^2(\theta) \\ &= \sec^{-2}(\theta). \end{aligned}$$

$$(c) \sec(\theta) - \cos(\theta) = \tan(\theta) \sin(\theta)$$

Solution :

$$\sec(\theta) - \cos(\theta) = \frac{1}{\cos(\theta)} - \cos(\theta)$$

$$= \frac{1}{\cos(\theta)} - \frac{\cos^2(\theta)}{\cos(\theta)}$$

$$= \frac{1 - \cos^2(\theta)}{\cos(\theta)}$$

$$= \frac{\sin^2(\theta)}{\cos(\theta)}$$

$$= \frac{\sin(\theta)}{\cos(\theta)} \sin(\theta)$$

$$= \tan(\theta) \sin(\theta)$$

$$(d) \log \tan(\theta) = \log \sin(\theta) - \log \cos(\theta)$$

Solution :

$$\log \tan(\theta) = \log \frac{\sin(\theta)}{\cos(\theta)} = \log \sin(\theta) - \log \cos(\theta).$$

8. Find the exact value of the following

$$(a) \sin(132\pi/3) \quad (0)$$

$$(b) \tan(3082\pi) \quad (0)$$

$$(c) \cot(3081\pi) \quad (0)$$

$$(d) \cos(264\pi/6) \quad (1)$$