Name: Dr. Mullikin

Test 2

Fall 2006

MATH 121 Section 02

October 6, 2006

Directions : You have 50 minutes to complete all 5 problems on this exam. There are a possible 100 points to be earned. You may not use your book, notes, or any graphing/programmable calculator. Please be sure to show all pertinent work. *An incorrect answer with no work will receive no credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

1. (20 points)
   Compute the derivatives of the following functions:

   (a) $f(x) = (x^2 + \sin(x))^{323}$
   (b) $g(x) = \sin(x)\cos(2x)$
   (c) $q(x) = x^2/(x^2+1)$
   (d) $F(x) = 5 + \sqrt{\sin(\sin(x))}$.

   Solution :

   (a)
$$f'(x) = 323(x^2 + \sin(x))^{322}(2x + \cos(x)).$$

   (b)
$$g'(x) = \cos(x)\cos(2x) + \sin(x)(-2\sin(2x))$$

   (c)
$$q'(x) = \frac{2x(x^2+1) - x^2(2x)}{(x^2+1)^2}$$

   (d)
$$F'(x) = \frac{1}{2}\left[\sin(\sin(x))\right]^{-1/2}\left[\cos(\sin(x))\right]\cos(x)$$

2. (20 points)
Let $r(x) = f(g(h(x)))$, where $h(1) = 2$, $g(2) = 3$, $h'(1) = 4$, $g'(2) = 5$, and $f'(3) = 0$. Find $r'(1)$.

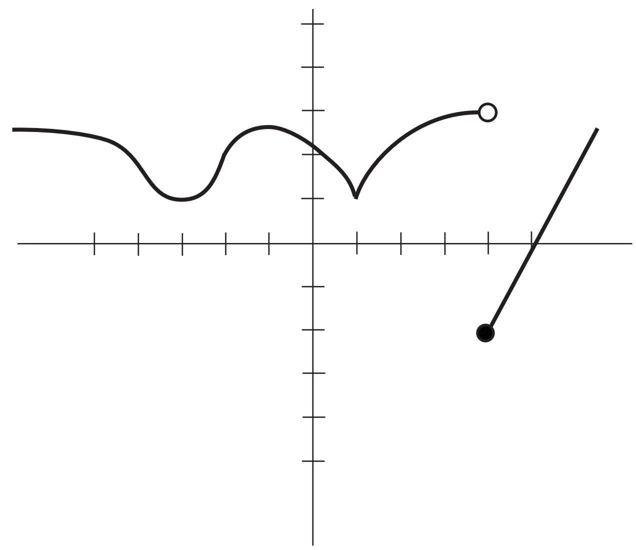
Solution : Using the chain rule we see that

$$r'(x) = f'(g(h(x)))g'(h(x))h'(x).$$

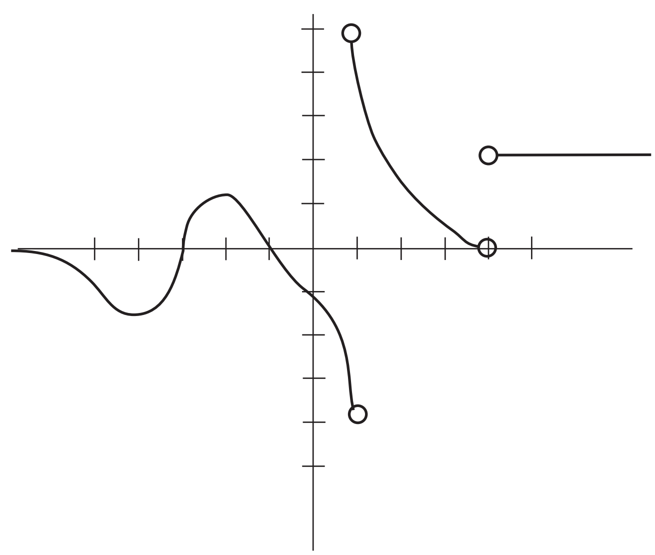
So,

$$\begin{aligned} r'(x) &= f'(g(h(1)))g'(h(1))h'(1) \\ &= f'(g(2))g'(2)4 \\ &= f'(3)(5)(4) \\ &= (0)(5)(4) \\ &= 0. \end{aligned}$$

3. (20 points)
Below is the graph of a function $f(x)$. Sketch the graph of the function $f'(x)$ where defined. If $f'(x)$ is not defined everywhere tell me where it is undefined and why it is undefined there.



Solution : The derivative does not exist at $x = 4$ since the function is not continuous there. It also does not exist at $x = 1$ since there is a corner point there.



4. (20 points)
Compute the equation of the tangent line to the curve given by the equation

$$x^2 + y^2 = 2x + 4y + 5$$

at the point $(0, 5)$.

Solution : We need only find the slope of the line. To do that we need to compute a derivative

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(2x + 4y + 5)$$

$$2x\frac{dx}{dx} + 2y\frac{dy}{dx} = 2\frac{dx}{dx} + 4\frac{dy}{dx}$$

$$2x + 2y\frac{dy}{dx} = 2 + 4\frac{dy}{dx}$$

$$2y\frac{dy}{dx} - 4\frac{dy}{dx} = 2 - 2x$$

$$(2y - 4)\frac{dy}{dx} = 2 - 2x$$

$$\frac{dy}{dx} = \frac{2 - 2x}{2y - 4}$$

$$\frac{dy}{dx} = \frac{1 - x}{y - 2}.$$

So, the slope at the point $(0, 5)$ will be $1/3$. The equation of the tangent line will then be

$$\ell(x) = \frac{1}{3}(x - 0) + 5.$$

5. (20 points)
   Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2 ft/sec. How fast is the area of the spill increasing when the radius of the spill is 60 ft?

   Solution : The equation we will be working with is the area equation $A(r) = \pi r^2$. Since the radius is increasing at a rate of 2 ft/sec we know that ${}^{dr}/_{dt} = 2$. Computing a derivative and using the fact that ${}^{dr}/_{dt} = 2$ and $r = 60$ we get

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(60)(2) = 240\pi.$$