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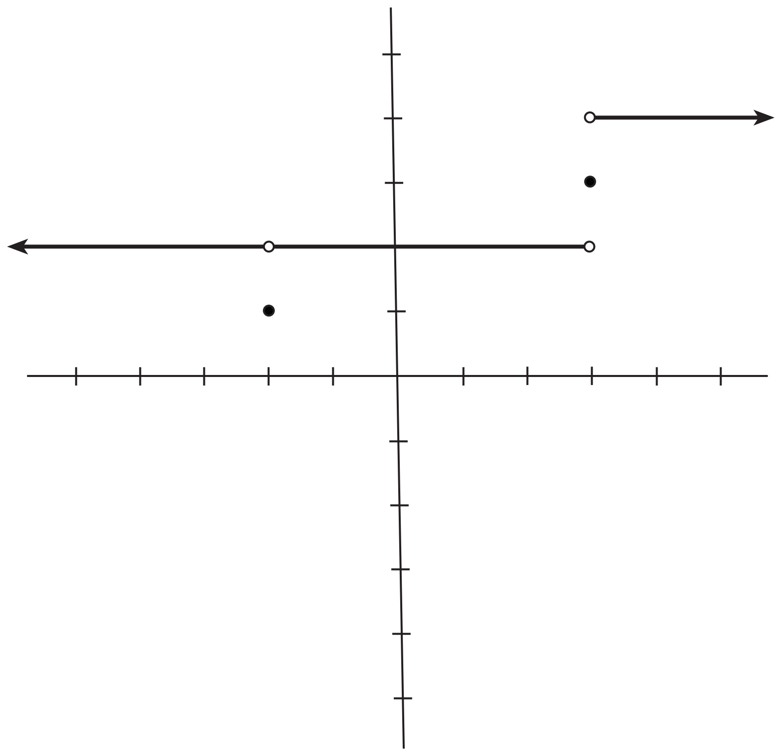
Test 1
Spring 2007
MTH121 Section 02
February 2, 2002

Directions : You have 50 minutes to complete all 5 problems on this exam. There are a possible 100 points to be earned. You may not use your book, notes, or any graphing/programmable calculator. Please be sure to show all pertinent work. *An incorrect answer with no work will receive no credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

1. (20 points)
   Sketch the graph of an example of a function f that satisfies all of the given conditions.

   $$\lim_{x\to 3^+} f(x) = 4,\ \lim_{x\to 3^-} f(x) = 2,\ \lim_{x\to -2} f(x) = 2,\ f(3) = 3,\ \text{and} f(-2) = 1.$$

   Solution :



2. (20 points)
Evaluate the limit, if it exists

a) $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$

b) $\lim_{x \to 7} \frac{\sqrt{x+2} - 3}{x - 7}$

Solution :

a)

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \to 2} \frac{(x+3)(x-2)}{x-2}$$

$$= \lim_{x \to 2} x + 3$$

$$= 5.$$

b)

$$\lim_{x \to 7} \frac{\sqrt{x+2} - 3}{x - 7} = \lim_{x \to 7} \frac{\sqrt{x+2} - 3}{x - 7} \left(\frac{\sqrt{x+2} + 3}{\sqrt{x+2} + 3} \right)$$

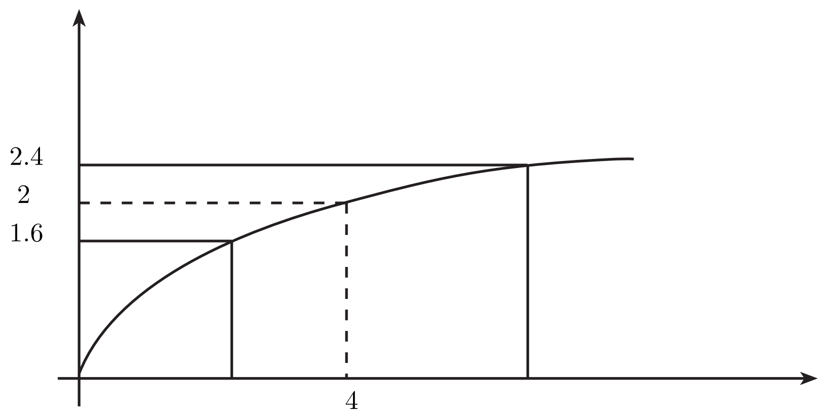
$$= \lim_{x \to 7} \frac{(x+2) - 9}{(x-7)(\sqrt{x+2} + 3)}$$

$$= \lim_{x \to 7} \frac{x - 7}{(x-7)(\sqrt{x+2} + 3)}$$

$$= \lim_{x \to 7} \frac{1}{\sqrt{x+2} + 3}$$

$$= \frac{1}{6}.$$

3. (20 points)
Use the graph of $f(x) = \sqrt{x}$ to find a number δ such that $|\sqrt{x} - 2| < 0.4$ whenever $|x - 4| < \delta$.



Solution : Let $\delta = \min\left\{(2.4)^2 - 4, 4 - (1.6)^2\right\} = \min\{1.76, 1.44\} = 1.44$.

4. (20 points)
If $f(x) = x^3 - x^2 + x$, show that there is a number c such that $f(c) = 10$.

Solution : The function $f(x)$ is a polynomial and is therefore continuous. Observe that $f(0) = 0 < 10$ and $f(10) = 910 > 10$. The intermediate value theorem guarantees that there is a point c in between $x = 0$ and $x = 10$ so that $f(c) = 10$.

5. (20 points)
Find the slope of the tangent to the curve $y = 2/(x+3)$ at the point where $x = a$ using the limit definition. You must use the limit definition if you wish to receive any credit.

Solution :

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{\frac{2}{a+h+3} - \frac{2}{a+3}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2(a+3) - 2(a+h+3)}{(a+3)(a+h+3)}}{h}$$

$$= \lim_{h \to 0} \frac{-2}{(a+3)(a+h+3)}$$

$$= -\frac{2}{(a+3)^2}$$