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Test 4
Spring 2007
MTH121 Section 02
April 19, 2007

Directions : You have 50 minutes to complete all 5 problems on this exam. There are a possible 100 points to be earned. You may not use your book, notes, or any graphing/programmable calculator. Please be sure to show all pertinent work. *An incorrect answer with no work will receive no credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

1. (35 points)
Consider the curve

$$y = x^3 + x.$$

(a) What is the domain of the curve?

(b) What are the x and y intercepts if any?

(c) Where is the curve increasing/decreasing?

(d) Identify all critical points as local maxima, minima, or neither.

(e) Where is the curve concave up/down?

(f) Identify all inflection points if any.

(g) Sketch the graph (on the next page).

Solution :

(a) The domain is $(-\infty, \infty)$.

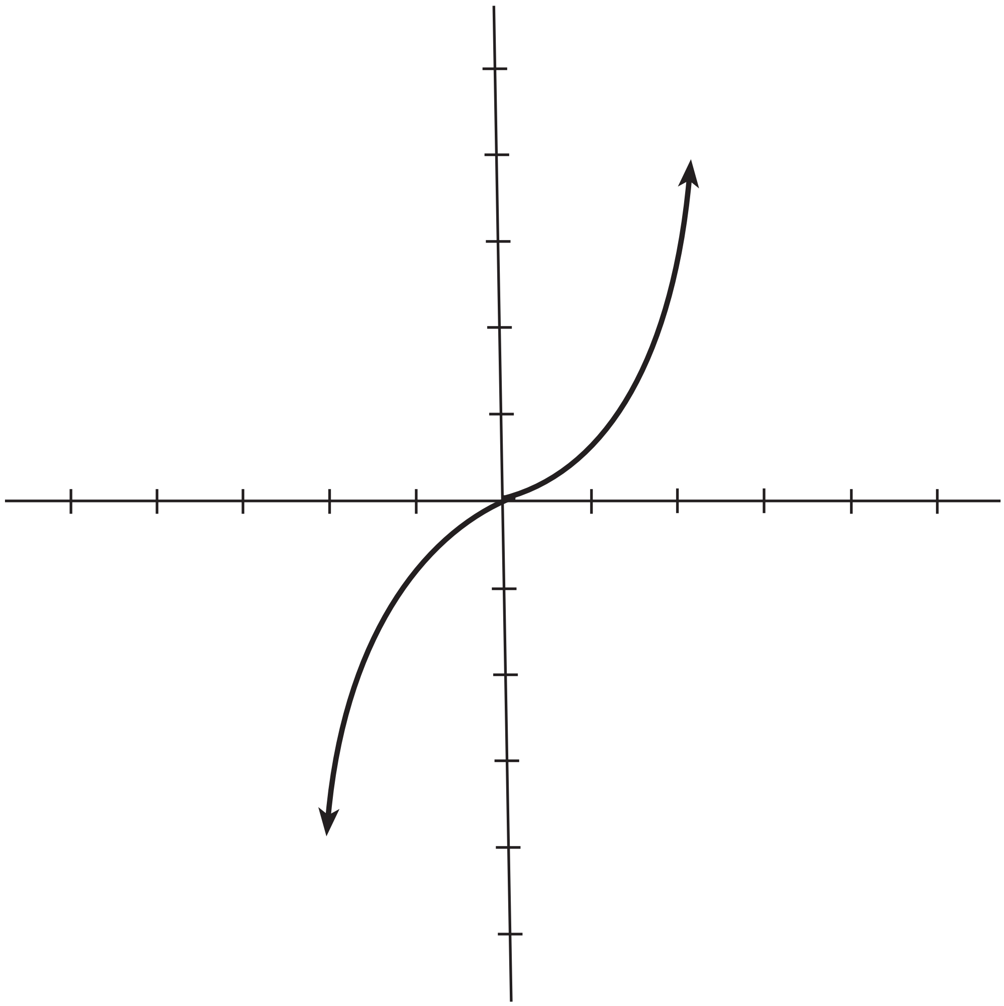
(b) The only x and y intercept is $(0, 0)$.

(c) The derivative is $y' = 3x^2+1$ which is always positive. So the function is increasing on its domain.

(d) There are no critical points since $3x^2 + 1$ is never zero or undefined.

(e) The second derivative is $y'' = 6x$. This is positive when $x > 0$ and it is negative when $x < 0$. So, the curve is concave up on the interval $(0, \infty)$ and concave down on the interval $(-\infty, 0)$.

(f) The concavity changes at the point $(0, 0)$, so this is the only inflection point.



2. (20 points)
If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

Solution : The volume of the box is $V = x^2 h$. Given that the amount of material to be used is 1200, we obtain the equation $x^2 + 4xh = 1200$ since the surface area of the box needs to be 1200. Solving this equation for h we see that $h = (1200 - x^2)/(4x)$ and so we now can write the volume as a function of x as

$$V(x) = x^2 \frac{1200 - x^2}{4x} = 300x - \frac{1}{4}x^3 \text{ where } x \text{ is in the interval } (0, \sqrt{1200})$$

We now look for critical points.

$$V'(x) = 300 - \frac{3}{4}x^2$$

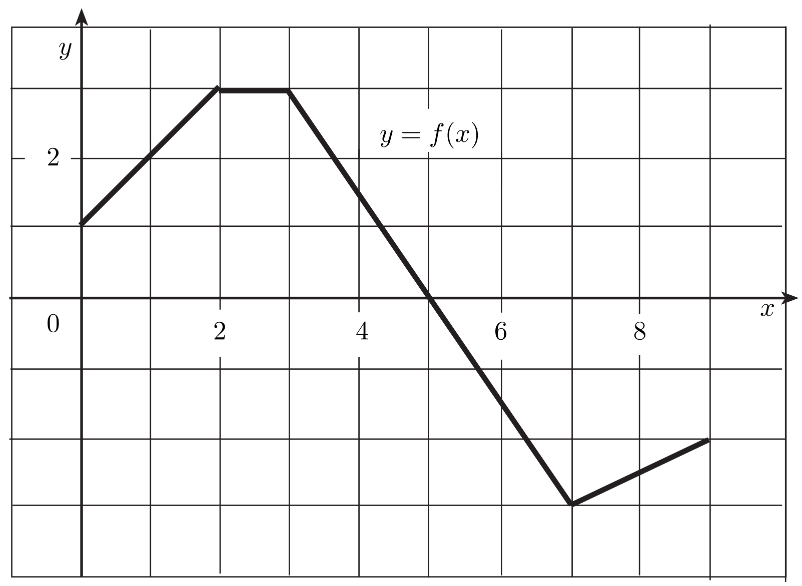
So, setting $V'(x)$ equal to zero we get

$$\begin{aligned} 300 - \frac{3}{4}x^2 &= 0 \\ \Rightarrow 300 &= \frac{3}{4}x^2 \\ \Rightarrow 1200 &= 3x^2 \\ \Rightarrow 400 &= x^2 \\ \Rightarrow 20 &= x. \end{aligned}$$

Using the second derivative test we compute $V''(x) = -\frac{3}{2}x$ which is negative for all positive values of x. So, the function $V(x)$ is concave down on the interval $(0, \sqrt{1200})$ so there is a max when $x = 20$. Therefore, the maximum volume is

$$V(20) = 300(20) - \frac{1}{4}(20)^3 = 4000.$$

3. (20 points)
The graph of f is shown. Evaluate the integral $\int_0^9 f(x)dx$ by interpreting it in terms of areas.



Solution : We simply add up the area of the region under the curve by breaking it up into rectangular and triangular pieces and adding up the areas keeping in mind that the area under the x-axis is negative. This gives us $10 + (-8) = 2$.

4. (20 points)
Use the Fundamental Theorem of Calculus to compute the following:

a) $\dfrac{d}{dx}\displaystyle\int_x^2 \cos(t^2)dt$ b) $\displaystyle\int_0^2 x(2+x^5)dx$

Solution :

(a)

$$\frac{d}{dx}\int_x^2 \cos(t^2)dt = \frac{d}{dx} - \int_2^x \cos(t^2)dt = \cos(x^2).$$

(b)

$$\int_0^2 x(2+x^5)dx = \int_0^2 2x + x^6 dx = x^2 + \frac{x^7}{7}\Big|_0^2 = 4 + \frac{128}{7} = \frac{156}{7}.$$