Name: Dr. Mullikin

Test 2
Spring 2007
MATH 122 Section 01
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Directions : You have 50 minutes to complete all 4 problems on this exam. There are a possible 100 points to be earned. You may not use your book or any graphing/programmable calculator. Please be sure to show all pertinent work. *An incorrect answer with no work will receive no credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

1. (20 points)
   Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

   (a) $\lim\limits_{t \to 0} \dfrac{e^t - 1}{t^3}$ (b) $\lim\limits_{x \to \infty} \dfrac{\ln(x)}{x}$

   Solution :

   a)
$$\lim_{t \to 0} \frac{e^t - 1}{t^3} \overset{\text{L}}{=} \lim_{t \to 0} \frac{e^t}{3t^2} \to \infty$$

   b)
$$\lim_{x \to \infty} \frac{\ln(x)}{x} \overset{\text{L}}{=} \lim_{x \to \infty} \frac{\left(\frac{1}{x}\right)}{1} = 0.$$

2. (10 points)
Use logarithmic differentiation to find the derivative of the function

$$y = x^{e^x}.$$

Solution : As instructed, consider the equation $\ln(y) = \ln\left(x^{e^x}\right)$. Using the properties of logarithms we rewrite $\ln(y) = e^x \ln(x)$. Then, computing the derivative of both sides we have

$$\frac{y'}{y} = e^x \ln(x) + e^x \frac{1}{x}.$$

Multiplying both sides of the last equation by $y = x^{e^x}$ gives us the desired result

$$y' = x^{e^x} \left(e^x \ln(x) + e^x \frac{1}{x} \right).$$

3. (20 points)
Find the derivative.

(a) $y = e^{\cosh(3x)}$

(b) $H(x) = (1 + x^2)\arctan(x)$

Solution :

a) $y' = e^{\cosh(3x)}(\sinh(3x))\,3$

b) $2x\arctan(x) + 1$

4. (50 points)
Evaluate the integral.

(a) $\displaystyle\int_1^2 \frac{dt}{8-3t}$

(b) $\displaystyle\int \frac{x+9}{x^2+9}dx$

(c) $\displaystyle\int \frac{\sinh(\sqrt{x})}{\sqrt{x}}dx$

(d) $\displaystyle\int x\cos(5x)dx$

(e) $\displaystyle\int \sec^2(x)\tan(x)dx$

Solution :

a) We make the u-substitution $u = 8-3t$. Then $-\frac{1}{3}du = dt$ and we have

$$\int_1^2 \frac{dt}{8-3t} = \int_5^2 -\frac{1}{3u}du = \frac{1}{3}\int_2^5 \frac{1}{u}du = \ln(5) - \ln(2).$$

b)

$$\begin{aligned}
\int \frac{x+9}{x^2+9}dx &= \int \frac{x}{x^2+9}dx + \int \frac{9}{x^2+9}dx \\
&= \int \frac{\frac{1}{2}}{u}du + \int \frac{9}{9\left(\frac{1}{9}x^2+1\right)}dx && \text{(Let } u = x^2+9\text{)} \\
&= \frac{1}{2}\ln(u) + \int \frac{1}{\frac{1}{9}x^2+1}dx \\
&= \frac{1}{2}\ln(x^2+9) + \int \frac{1}{\left(\frac{x}{3}\right)^2+1}dx \\
&= \frac{1}{2}\ln(x^2+9) + \int \frac{3}{u^2+1}du && \text{(Let } u = \frac{x}{3}\text{)} \\
&= \frac{1}{2}\ln(x^2+9) + 3\arctan(u) + C \\
&= \frac{1}{2}\ln(x^2+9) + 3\arctan\left(\frac{x}{3}\right) + C
\end{aligned}$$

c)

$$\begin{aligned}
\int \frac{\sinh(\sqrt{x})}{\sqrt{x}}dx &= \int 2\sinh(u)du && \text{(Let } u = \sqrt{x} \text{ so } 2du = \frac{1}{\sqrt{x}}dx\text{)} \\
&= 2\cosh(u) + C \\
&= 2\cosh(\sqrt{x}) + C
\end{aligned}$$

d) We use integration by parts with $u = x$ and $dv = \cos(5x)$.

$$\int x\cos(5x)dx = s\frac{\sin(5x)}{5} - \int\frac{\sin(5x)}{5}dx = \frac{x\sin(5x)}{5} + \frac{\cos(5x)}{25} + C$$

e)

$$\begin{aligned}\int \sec^2(x)\tan(x)dx &= \int udu \qquad\qquad \text{(Let } u = \sec(x)\text{)}\\ &= \frac{u^2}{2} + C\\ &= \frac{\sec^2(x)}{2} + C\end{aligned}$$