MATH 462
Midterm Examination

You will have until Friday, March 7th to complete the following problems. You may use your calculus book, Shifrin's book, your brain, and my brain. Please do not use any other books or anyone else's brain.

1. Let $f\begin{pmatrix} x \\ y \end{pmatrix} = 2x^2 + xy - y^2$ and let $P = \{(x, y) \in \mathbb{R}^2 : 1 \leq 2x - y \leq 3, -2 \leq x + y \leq 0\}$. Compute the integral
$$\int_P f\begin{pmatrix} x \\ y \end{pmatrix} dA$$
by turning it into an integral of the form
$$\int_\Omega (f \circ g)\begin{pmatrix} u \\ v \end{pmatrix} |\det Dg|\, dA$$

2. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$ in terms of the (nonzero) constants a, b, and c.

3. Find the volume of the "pillow" $\rho = \sin(\theta)$, $0 \leq \theta \leq \pi$. (Use spherical coordinates).

4. Find the distance from $(1, 1, 1)$ to the line of intersection of the planes $2x + y - z = 1$ and $x - y + z = 2$. [*Hint:* Let (x, y, z) be a point on the line and minimize the square of its distance from $(1, 1, 1)$.]

5. Let $\Omega \subset \mathbb{R}^n$ be a region. Let $f : \Omega \longrightarrow \mathbb{R}$ and $g : \Omega \longrightarrow \mathbb{R}$ be functions that are integrable on Ω that differ only on the boundary of Ω. Show that
$$\int_\Omega f\, dV = \int_\Omega g\, dV.$$