Name:____________________

Final Examination (kinda)
Summer 2007
MTH481 Section 01
July 14-20, 2007

Directions : This is the work-at-home version of the final exam. There are 4 problems and they count for 60% of your final exam grade. You may use your book, each other, and me. **Please do not use any other sources.**

1) Suppose $f : (X, \mathbf{T}) \longrightarrow (Y, \mathbf{S})$ is a continuous function from one topological space to another. Prove the following.

   i) If $\mathbf{U}$ is a topology for Y such that $\mathbf{U} \subset \mathbf{S}$, then $f : (X, \mathbf{T}) \longrightarrow (Y, \mathbf{U})$ is continuous.

   ii) If $\mathbf{V}$ is a topology for X such that $\mathbf{T} \subset \mathbf{V}$, then $f : (X, \mathbf{V}) \longrightarrow (Y, \mathbf{S})$ is continuous.

b) If X, Y, and Z are topological spaces, X and Y are homeomorphic, and Y and Z are homeomorphic, then X and Z are homeomorphic. (We've been using this in class but we haven't actually proven it yet. Have at it.)

III) Suppose $[a, b]$ is an interval of real numbers with the usual topology, and let $f : [a, b] \longrightarrow \mathbb{R}$, again with the usual topology, be continuous. Prove that if c is a real number between $f(a)$ and $f(b)$, then there is an $x \in [a, b]$ such that $f(x) = c$. (This is the celebrated Intermediate Value Theorem.)

Four) We need a couple of definitions for this one.

Definition : *If a and b are points in a topological space X, a* ***path*** *in X from a to b is a continuous function $f : [0, 1] \longrightarrow X$ such that $f(0) = a$ and $f(1) = b$, where the interval $[0, 1]$ has the usual topology.*

Definition : *A topological space is* ***path-connected*** *if for every a and b in X there is a path in X from a to b.*

Suppose x, y, and z are points in a topological space X. Suppose that there is a path in X from x to y and a path in X from y to z. Prove there is a path in X from x to z.

Bonus) Prove or disprove. If S is a path-connected subset of a space X, and $S \subset K \subset \text{cl}(S)$, then K is path-connected.