(1 point) Name:_____

Test 1
Spring 2005
CSCI/MATH 2610
February 10, 2005

Directions : You have 75 minutes to complete all 6 problems on this exam. There are a possible 110 points to be earned with 10 extra credit points built in. You may not use your book or any notes. Please be sure to show all pertinent work. *An answer with no work will receive very little credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

1. (25 points)
   Answer each of the following questions.

   (a) What is the definition of an injective function.

   (b) What is the definition of a surjective function.

   (c) What is the negation of the statement $\exists x \exists y (P(x,y) \lor Q(x,y))$?

   (d) Let A and B be sets. Define $A \times B$.

   (e) Let $A = \{\emptyset, \{a\}, \heartsuit\}$ what is $\mathcal{P}(A)$? List the elements. ($\mathcal{P}(A)$ denotes the power set of A.)

2. (15 points)

Let P, Q, and R be propositions. Prove or disprove that

$$[((P \vee Q) \wedge \sim P) \to Q] \vee (R \to \sim R) \equiv \mathbf{T}.$$

3. (18 points)
   Use Venn diagrams to sketch the following sets.

   (a) $A \cap \left(\overline{B - C} \right)$

   (b) $[A \cup B \cup C) - ((A \cap B) \cup (A \cap C) \cup (B \cap C))] \cup (A \cap B \cap C)$

4. (15 points)

   Let n be an integer. Prove that if $5n + 3$ is odd, then n is even.

5. (16 points)

   Define a function $f : \mathbb{Z}^+ \longrightarrow \mathbb{Z}^+$ that is

   (a) neither injective or surjective.

   (b) injective but not surjective.

   (c) surjective but not injective.

   (d) a bijection other than the identity function $f(x) = x$.

6. (20 points)

Let $A = \{1, 2, 3, \ldots\}$ and $B = \{-1, -2, -3, \ldots\}$. Prove $|A \cup B| = |\mathbb{Z}^+|$.