MATH 2610
Discrete Mathematics for Computer Science
Tuesday February, 16 2005

(1) Find a formula for

$$\sum_{i=1}^{n} \frac{1}{2^i} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$$

by examining the values of this expression for small values of n.

(2) Use mathematical induction to prove your claim for the previous problem.

(3) Prove using mathematical induction that

$$\sum_{i=1}^{n} i(i+1) = 1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = n(n+1)(n+2)/3$$

whenever $n \in \mathbb{Z}^+$.

(4) Show that

$$\sum_{i=1}^{n} (-1)^{i-1} i^2 = 1^2 - 2^2 + 3^2 - \cdots + (-1)^{n-1} n^2 = (-1)^{n-1} n(n+1)/2$$

whenever $n \in \mathbb{Z}^+$.

(5) Show that $n^2 - 1$ is divisible by 8 whenever n is an odd positive integer. Be careful about the $P(k) \to P(k+1)$ part! We only care about *odd* positive integers.

(6) Prove that a set with n elements has $n(n-1)/2$ subsets containing exactly two elements whenever n is an integer greater than or equal to 2.

(7) Prove that a set with n elements has $n(n-1)(n-2)/6$ subsets containing exactly three elements whenever n is an integer greater than or equal to 3.

(8) What is wrong with this "proof"?

"*Theorem*" For every positive integer n, if x and y are positive integers with $\max(x, y) = n$, then $x = y$.

"*Proof*" Suppose that n = 1. If $\max(x, y) = 1$ and x and y are positive integers, we have $x = 1$ and $y = 1$.

Let k be a positive integer. Assume that whenever $\max(x, y) = k$ and x and y are positive integers, then $x = y$. Now let $\max(x, y) = k + 1$, where x and y are positive integers. Then $\max(x - 1, y - 1) = k$, so by the inductive hypothesis, $x - 1 = y - 1$. It follows that $x - y$, completing the inductive step. $\square$